77-FM-41

JSC-12988

Ground Tracking Data Program Document

(NASA-TM-74866) SHUTTLE PROGRAM: GROUND TRACKING DATA PROGRAM DOCUMENT SHUTTLE OFT LAUNCH/LANDING (NASA) 97 p HC A05/MF A01 CSCL 22A

N77-30167

Unclas
G3/16 45634

Shuttle OFT Launch/Landing

Mission Planning and Analysis Division

August 1977

National Aeronautics and
Space Administration

Lyndon B. Johnson Space Center
Houston, Texas

77-FM-41 JSC-12988

SHUTTLE PROGRAM

GROUND TRACKING DATA PROGRAM DOCUMENT
SHUTTLE OFT LAUNCH/LANDING

By William M. Lear, TRW Systems Group

JSC Task Monitor: Paul H. Mitchell FM8

Approved: ______________________
Emil R. Schiesser, Chief
Mathematical Physics Branch

Approved: ______________________
Ronald L. Berry, Chief
Mission Planning and Analysis Division

National Aeronautics and Space Administration

Lyndon B. Johnson Space Center

Mission Planning and Analysis Division

Houston, Texas

August 1977

PREFACE

William M. Lear of TRW, under contract to JSC, derived the formulation of a Kalman filter and all related transformation matrices and measurement equations that are suitable for ground navigation trajectory determination for the space shuttle. This work was first presented to JSC in the TRW Technical Report No. 30759-6002-RU-00 (ref. 3) and is now being converted to this JSC internal note by Paul Mitchell of the Mathematical Physics Branch. This material includes the following: mathematical equations for processing goundtracking data during shuttle ascent and entry, typical error model statistics and other constants, and the Fortran listings of a complete bench program that includes most of the important features and formulation of the high-speed trajectory determination processor adopted by JSC for the real-time space shuttle ascent and entry phases. This TRW bench program has been used to perform analysis designed to give answers to be used in shuttle planning and design for ascent and entry pertaining to ground-based navigation. It can process data from three groundtracking stations, two C-band radars, and one S-band radar with a total of 10 measurements with common time tags. The measurements are C-band 1 range, azimuth and elevation; C-band 2 range, azimuth and elevation; and S-band range, Doppler and two angles from either a 30-ft station or an 85-ft station.

CONTENTS

GROUND TRACKING DATA PROGRAM DOCUMENT

SHUTTLE OFT LAUNCH/LANDING

By William M. Lear

1.0 INTRODUCTION

This document gives the equations for processing ground tracking data during a Space Shuttle ascent or entry, or any nonfree flight phase of a shuttle mission. The resulting computer program will process data from up to three stations simultaneously: C-band station number 1, C-band station number 2, and an S-band station. The C-band data consists of range, azimuth, and elevation angle measurements. The S-band data consists of range, two angles, and integrated Doppler data in the form of cycle counts.

A nineteen element state vector is used in a Kalman filter to process the measurements. The first nine elements of the state vector are:

$\underline{R}_{EF}$ = position of the shuttle in earth-fixed coordinates,

$\dot{\underline{R}}_{EF}$ = velocity of the shuttle in earth-fixed coordinates,

$\ddot{\underline{R}}_{EF}$ = acceleration of the shuttle in earth-fixed coordinates.

The acceleration components of the shuttle are taken to be independent exponentially-correlated random variables. No gravitational acceleration is included. Typical statistics for these random variables are a time constant of 40 seconds and a standard deviation of 6 m/sec^2.

The next nine elements of the state vector are the measurement bias errors associated with range and two angles for each tracking station. The biases are all modeled as exponentially-correlated random variables with a typical time constant of 108 seconds. All time constants are taken to be the same for all nine state variables. This simplifies the logic in propagating the state error covariance matrix ahead in time.

The nineteenth element of the state vector is the integration constant associated with the integrated Doppler measurements. It is modeled as a random walk to account for range rate errors adding to the measurement.

2.0 STATE VECTOR ELEMENTS

In more detail, the elements of the state vector are given by

$$\left.\begin{aligned} x_1 &= X_{EF} \\ x_2 &= Y_{EF} \\ x_3 &= Z_{EF} \end{aligned}\right\} \underline{R}_{EF}, \text{ position of the shuttle in earth-fixed coordinates.}$$

$$\left.\begin{aligned} x_4 &= \dot{X}_{EF} \\ x_5 &= \dot{Y}_{EF} \\ x_6 &= \dot{Z}_{EF} \end{aligned}\right\} \underline{\dot{R}}_{EF}, \text{ velocity of the shuttle in earth-fixed coordinates.}$$

$$\left.\begin{aligned} x_7 &= \ddot{X}_{EF} \\ x_8 &= \ddot{Y}_{EF} \\ x_9 &= \ddot{Z}_{EF} \end{aligned}\right\} \underline{\ddot{R}}_{EF}, \text{ acceleration of the shuttle in earth fixed coordinates.}$$

$x_{10} = B_{\rho 1}$, range bias for first C-band station.

$x_{11} = B_{A1}$, azimuth bias for first C-band station, radians.

$x_{12} = B_{E1}$, elevation bias for first C-band station, radians.

$x_{13} = B_{\rho 2}$, range bias for second C-band station.

$x_{14} = B_{A2}$, azimuth bias for second C-band station, radians.

$x_{15} = B_{E2}$, elevation bias for second C-band station, radians.

$x_{16} = B_{\rho 3}$, range bias for S-band station.

$x_{17} = B_{\alpha X}$, X angle bias for S-band station, radians.

$x_{18} = B_{\alpha Y}$, Y angle bias for S-band station, radians

$x_{19} = I_D$, Doppler integration constant for S-band station, cycles.

3.0 STATE VECTOR DYNAMICS

State vector elements 7 through 18 are exponentially correlated random variables, assumed constant over the integration step. The dynamical equation for an exponentially correlated random variable, ε, is

$$\varepsilon_i = a\varepsilon_{i-1} + \sigma_\varepsilon \sqrt{1 - a^2}\, n_i$$

$$a = \exp\left(-\Delta T/\tau_\varepsilon\right)$$

where

$$E[n_i] = 0$$

$$E[n_i n_j] = 0 \qquad i \neq j$$

$$= 1 \qquad i = j$$

and where τ_ε is the time constant associated with ε: τ_ε small - a rapidly changing random variable, τ_ε large - a slowly changing random variable. σ_ε is the standard deviation of ε. Since the best estimate of n_i is zero, the filter's best estimate of ε is propagated ahead by

$$\varepsilon_i = \exp(-\Delta T/\tau_\varepsilon)\ \varepsilon_{i-1}$$

with the term $\sigma_\varepsilon\sqrt{1 - a^2}\, n_i$ appearing in the state noise vector, and its variance appearing in the state noise covariance matrix.

Thus, in engineering notation, the first nine elements of the filter's estimated state vector are integrated ahead with

$$\underline{R}_{EF,i} = \underline{R}_{EF,i-1} + \dot{\underline{R}}_{EF,i-1}\Delta T + \ddot{\underline{R}}_{EF,i-1}\,\Delta T^2/2$$

$$\dot{\underline{R}}_{EF,i} = \dot{\underline{R}}_{EF,i-1} + \ddot{\underline{R}}_{EF,i-1}\,\Delta T$$

$$\ddot{\underline{R}}_{EF,i} = \exp\,(-\Delta T/\tau_a)\,\ddot{\underline{R}}_{EF,i-1}$$

In component form, using state vector notation, the equations for propagating the state vector ahead in time are

$$x_{1,i} = x_{1,i-1} + x_{4,i-1}\,\Delta T + x_{7,i-1}\,\Delta T^2/2$$

$$x_{2,i} = x_{2,i-1} + x_{5,i-1}\,\Delta T + x_{8,i-1}\,\Delta T^2/2$$

$$x_{3,i} = x_{3,i-1} + x_{6,i-1}\,\Delta T + x_{9,i-1}\,\Delta T^2/2$$

$$x_{4,i} = x_{4,i-1} + x_{7,i-1}\,\Delta T$$

$$x_{5,i} = x_{5,i-1} + x_{8,i-1}\,\Delta T$$

$$x_{6,i} = x_{6,i-1} + x_{9,i-1}\,\Delta T$$

$$x_{7,i} = \exp\,(-\Delta T/\tau_a)\,x_{7,i-1}$$

$$x_{8,i} = \exp\,(-\Delta T/\tau_a)\,x_{8,i-1}$$

$$x_{9,i} = \exp(-\Delta T/\tau_a)\, x_{9,i-1}$$

$$x_{10,i} = \exp(-\Delta T/\tau_B)\, x_{10,i-1}$$

$$x_{11,i} = \exp(-\Delta T/\tau_B)\, x_{11,i-1}$$

$$\vdots$$

$$x_{18,i} = \exp(-\Delta T/\tau_B)\, x_{18,i-1}$$

$$x_{19,i} = x_{19,i-1}$$

We see that the dynamical equations are linear. That is, we can write

$$\underline{x}_i = \emptyset\, x_{i-1}$$

where $\emptyset$ is the state transition matrix. Let

$$P_2 = \Delta T \qquad P_3 = \Delta T^2/2$$

$$P_{10} = \exp(-\Delta T/\tau_a) \qquad PS_{71} = \exp(-\Delta T/\tau_B)$$

Then the state transition matrix can be written as shown below.

$$
\emptyset = \begin{bmatrix}
1 & 0 & 0 & P_2 & 0 & 0 & P_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & P_2 & 0 & 0 & P_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & P_2 & 0 & 0 & P_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & P_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & P_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & P_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & P_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & PS_{71} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & PS_{71} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & PS_{71} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & PS_{71} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & PS_{71} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & PS_{71} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & PS_{71} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & PS_{71} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & PS_{71} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

4.0 THE STATE NOISE COVARIANCE MATRIX

As mentioned in the previous section, all elements of the state vector which are exponentially correlated random variables contribute an element to the state noise vector of

$$\varepsilon_i = a\,\varepsilon_{i-1} + \underline{\sigma_\varepsilon \sqrt{1 - a^2}\,\eta_i}$$

where $a = \exp(-\Delta T/\tau_\varepsilon)$

$$E[\eta_i] = 0$$

$$E[\eta_i\,\eta_j] = 0 \qquad i \neq j$$

$$= 1 \qquad i = j$$

The Doppler integration constant is modeled differently. Its equation is

$$I_{D,i} = I_{D,i-1} + \sigma_\omega \eta_{\omega,i} \Delta T$$

where $E[\eta_{\omega,i}] = 0$

$$E[\eta_{\omega,i}\,\eta_{\omega,j}] = 0 \qquad i \neq j$$

$$= 1 \qquad i = j$$

where $\sigma_\omega \eta_{\omega,i} \Delta T$ is the state noise term, and is due to integrating the Doppler rate bias error, $\sigma_\omega \eta_{\omega,i}$. In order to make the variance of I_D independent of ΔT, we must make $\sigma_\omega \sim 1/\sqrt{\Delta T}$. This is easily seen by writing out

$$I_{D,i} = I_{D,0} + \sigma_\omega(\eta_{\omega,1} + \eta_{\omega,2} + \eta_{\omega,3} + \cdots + \eta_{\omega,i})\,\Delta T$$

$$E[I^2_{D,i}] = E[I^2_{D,0}] + \sigma^2_\omega\, i\, \Delta T^2$$

$$= E[I_{D,0}^2] + (\sigma_\omega^2 \Delta T)(i \Delta T)$$

where $i\Delta T = T$, the total integration time. To make $E[I_{D,i}^2]$ independent of ΔT we must set

$$K = \sigma_\omega^2 \Delta T$$

or

$$\sigma_\omega = \sqrt{K/\Delta T}$$

Thus

$$I_{D,i} = I_{D,i-1} + \sqrt{K/\Delta T}\, n_{\omega,i}\, \Delta T$$

Thus the state noise vector is given by

$$\underline{s} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \sigma_a\sqrt{1-\exp(-2\Delta T/\tau_a)}\; n_{a1} \\ \sigma_a\sqrt{1-\exp(-2\Delta T/\tau_a)}\; n_{a2} \\ \sigma_a\sqrt{1\ \exp(-2\Delta T/\tau_a)}\; n_{a3} \\ \sigma_{\rho 1}\sqrt{1-\exp(-2\Delta T/\tau_B)}\; n_{\rho 1} \\ \sigma_{A1}\sqrt{1-\exp(-2\Delta T/\tau_B)}\; n_{A1} \\ \sigma_{E1}\sqrt{1-\exp(-2\Delta T/\tau_B)}\; n_{E1} \\ \sigma_{\rho 2}\sqrt{1-\exp(-2\Delta T/\tau_B)}\; n_{\rho 2} \\ \sigma_{A2}\sqrt{1-\exp(-2\Delta T/\tau_B)}\; n_{A2} \\ \sigma_{E2}\sqrt{1-\exp(-2\Delta T/\tau_B)}\; n_{E2} \\ \sigma_{\rho 3}\sqrt{1-\exp(-2\Delta T/\tau_B)}\; n_{\rho 3} \\ \sigma_{\alpha X}\sqrt{1-\exp(-2\Delta T/\tau_B)}\; n_{\alpha X} \\ \sigma_{\alpha Y}\sqrt{1-\exp(-2\Delta T/\tau_B)}\; n_{\alpha Y} \\ \sqrt{K\Delta T}\; n_{\omega} \end{bmatrix}$$

The state noise covariance matrix is given by $S = E[\underline{s}\,\underline{s}^T]$. Let

$$P_{12} = \sigma_a^2 \left[1-\exp(-2\Delta T/\tau_a)\right]$$

$$PS_{72} = 1 - \exp(-2\Delta T/\tau_B)$$

Then the diagonal state noise covariance matrix has the diagonal elements of

$$S_{1,1} = S_{2,2} = \cdots = S_{6,6} = 0$$

$$S_{7,7} = S_{8,8} = S_{9,9} = P_{12}$$

$$S_{10,10} = \sigma_{\rho 1}^2\, PS_{72}$$

$$S_{11,11} = \sigma_{A1}^2\, PS_{72}$$

$$S_{12,12} = \sigma_{E1}^2\, PS_{72}$$

$$S_{13,13} = \sigma_{\rho 2}^2\, PS_{72}$$

$$S_{14,14} = \sigma_{A2}^2\, PS_{72}$$

$$S_{15,15} = \sigma_{E2}^2\, PS_{72}$$

$$S_{16,16} = \sigma_{\rho 3}^2\, PS_{72}$$

$$S_{17,17} = \sigma_{\alpha X}^2\, PS_{72}$$

$$S_{18,18} = \sigma_{\alpha Y}^2\, PS_{72}$$

$$S_{19,19} = K\Delta T = PS_{69}$$

The value of K for the Space Shuttle is chosen in the following manner. From before we had

$$E[I_{D,i}^2] = E[I_{D,0}^2] + (\sigma_\omega^2 \Delta T)(i\Delta T)$$

$$= E[I_{D,0}^2] + KT$$

where T is the total time interval. For Apollo the rate bias error was a time-wise correlated random variable whose standard deviation was 0.005 cycles/sec. We used a time constant of 800 seconds for this random variable, even though the actual time constant was less.

In Apollo the Doppler frequency was multiplied by 1, for the Shuttle the Doppler frequency is multiplied by 1000, now giving an apparent rate bias error of 5 cycles/sec. Assume now a constant rate bias error acting over 400 seconds, then

$$K400 = 5^2 \cdot 400^2$$

$$K = 10000 \text{ cycles}^2/\text{sec}$$

That is, this value of K will add a variance of $5^2 \cdot 400^2$ cycles^2 over a 400 second period to the pre-existing variance of the Doppler integration constant. For the S-band frequencies used, the conversion factor from meters to cycles is about 15,200 cycles/meter. Thus $5^2 \cdot 400^2 = 0.13^2$ meters^2, not much.

5.0 MEASUREMENT EQUATIONS

5.1 INTRODUCTION

The equations in this section are taken from reference 1. This reference also contains the necessary equations for the C-band stations.

The subscript EF stands for earth-fixed coordinates whose origin is at the center of the earth. The Z_{EF} axis is the earth's axis of rotation, Y_{EF} goes through the Greenwich meridian at 0 degrees longitude, and Y_{EF} completes the right-handed coordinate system. Position of the Space Shuttle vehicle is denoted by

$\underline{R}_{V,EF}$ = position of vehicle in EF coordinates, the first 3 elements of the state vector.

Position of the tracking antenna is denoted by

$\underline{R}_{A,EF}$ = position of the antenna in EF coordinates.

The position of the vehicle with respect to the antenna is denoted by

$$\underline{R}_{V/A,EF} = \underline{R}_{V,EF} - \underline{R}_{A,EF}$$

The topodetic (subscript TOP) coordinate system has its origin at the center of the earth. It is rigidly attached to the earth and is thus rotating with the earth. The direction of its axes are referenced to EAST, NORTH, and UP at a specific antenna (station) on the surface of the earth. The Z_{TOP} axis parallel to the local vertical (normal to the reference ellipsoid) at the antenna. X_{TOP} points east and Y_{TOP} points north. TOP coordinates are obtained from EF coordinates by the constant coordinate transformation matrix, T5.

$$\underline{R}_{TOP} = T5\ \underline{R}_{EF}$$

The elements of T5 are functions of a particular antenna location. Let ϕ be the geodetic latitude of the antenna and λ the longitude (+ east). Then the

first row of T5 is a unit vector in EF coordinates in the east direction.

$$T5_{11} = -\sin\lambda$$

$$T5_{12} = \cos\lambda$$

$$T5_{13} = 0$$

The second row of T5 is a unit vector in EF coordinates in the north direction.

$$T5_{21} = -\sin\phi\cos\lambda$$

$$T5_{22} = -\sin\phi\sin\lambda$$

$$T5_{23} = \cos\phi$$

The third row of T5 is a unit vector in EF coordinates normal to the surface of the reference ellipsoid (the local vertical direction).

$$T5_{31} = \cos\phi\cos\lambda$$

$$T5_{32} = \cos\phi\sin\lambda$$

$$T5_{33} = \sin\phi$$

The state vector $\underline{R}_{V,EF}$ is in earth-fixed coordinates. Partial derivatives of the measurements taken in topodetic coordinates must be converted back to EF coordinates. That is, if y^* is the measurement, we use

$$\frac{\partial y^*}{\partial \underline{R}_{V,EF}} = \frac{\partial y^*}{\partial \underline{R}_{V/A,TOP}} \underbrace{\frac{\partial \underline{R}_{V/A,TOP}}{\partial \underline{R}_{V/A,EF}}}_{T5} \underbrace{\frac{\partial \underline{R}_{V/A,EF}}{\partial \underline{R}_{V,EF}}}_{I}$$

where I is the 3 by 3 identity matrix.

The measurement refraction correction calculations are shown below. The variables appearing in the equations must be generated by the estimated state vector. Let

$N_o = n_o - 1$ = index of refraction minus 1, the refraction modulus at at the antenna site.

H_S = atmospheric scale height. N_o and H_S are supplied by meteorologists at the tracking site.

R_o = 6 378 165 meters. Its value is not critical.

$\rho = |\underline{R}_{V/A}|$, geometric (straight line) value of range. Always less than the measured value of range, ρ_M.

E = geometric or true elevation angle. Always less than the measured elevation angle, E_M.

$\Delta\rho = \rho_M - \rho$, refraction correction for range.

$\Delta E = E_M - E$, refraction correction for elevation angle.

H = altitude above the tracking site.

H^* = height of a spherical slab atmosphere.

The refraction corrections are given by

$$\text{IF}(\rho < .02\ R_o \text{ AND } E < .6 \text{ deg})\ E = .6 \text{ degrees}$$

$$H = \sqrt{R_o^2 + \rho^2 + 2\rho\, R_o \sin E} - R_o$$

$$H^* = [1 - \exp(-H/H_S)]\, H_S$$

$$K = 2.7 \cdot 10^7\, N_o^{1.5} \frac{H^*}{R_o} (\cos E)^{1.4 \cdot 10^6\, N_o}$$

$$\Delta\rho = N_o R_o \left[\sqrt{\sin^2 E + 2H^*/R_o} - \sin E\right] (1 - K)$$

$$\Delta E = \frac{\Delta\rho \cos E}{N_o H^*} [N_o - \Delta\rho/\rho]$$

Note that for short ranges, of ρ less than .02 earth radii, the elevation angle is restricted to being greater than .6 degree. This is because the equations become inaccurate at short ranges and low elevation angles.

5.2 TWO-WAY RANGE MEASUREMENTS

Two-way range measurements are made by sending out a "ping" from the radar and measuring the round-trip time of the ping from the radar, to the vehicle, and back to the radar again. The round-trip time divided by the speed of light in a vacuum is the measured range, $\rho_M = \rho^*$. If the vehicle has a transponder to amplify and return the ping, the transponder may introduce a delay time long enough to affect the range measurement. This delay time would look like a bias adding to the range measurement. Note that there is no delay time for skin tracking. Since skin tracking will be used with the Space Shuttle, we will neglect this delay time in our equations. The actual range measurement is given by

$$\rho^* = \rho - \rho\dot{\rho}/c + \Delta\rho + B_\rho + q$$

where

$\rho = |\underline{R}_{V/A,EF}|$

$\rho\dot{\rho}/c$ = speed of light correction

$= \underline{R}^T_{V/A,EF}\ \dot{\underline{R}}_{V/A,EF}/c$ (T stands for transpose.)

c = speed of light in vacuum

$\Delta\rho$ = previously defined refraction correction

B_ρ = range bias, an exponentially correlated rangom variable, due to instrumentation errors and errors in calculating $\Delta\rho$.

q = uncorrelated noise

Typical skin track statistics are

$$\tau_{B\rho} = 108 \text{ seconds}$$

For a TPQ-18 or FPQ-6 C-band radar

$$\sigma_{B\rho} = 12 \text{ meters}$$

$$\sigma_q = 6 \text{ meters}$$

For an FPS-16 C-band radar, and for an S-band radar using a transponder on the vehicle

$$\sigma_{B\rho} = 18 \text{ meters}$$

$$\sigma_q = 9 \text{ meters}$$

Neglecting the speed of light and the refraction correction terms, the partial derivatives of ρ^* with respect to the elements in the state vector are

$$\frac{\partial \rho^*}{\partial X_{V,EF}} = X_{V/A,EF} / |\underline{R}_{V/A,EF}|$$

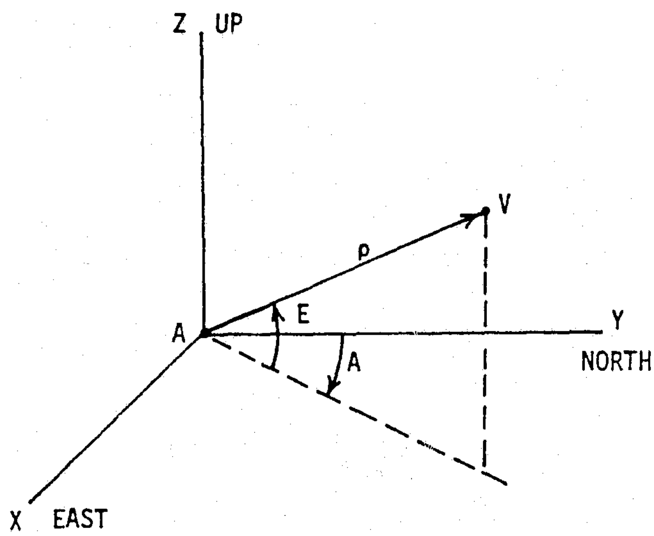
$$\frac{\partial \rho^*}{\partial Y_{V,EF}} = Y_{V/A,EF} / |\underline{R}_{V/A,EF}|$$

$$\frac{\partial \rho^*}{\partial Z_{V,EF}} = Z_{V/A,EF} / |\underline{R}_{V/A,EF}|$$

$$\frac{\partial \rho^*}{\partial B_\rho} = 1$$

5.3 AZIMUTH AND ELEVATION ANGLE MEASUREMENT

Azimuth angle, A, and elevation angle, E are measured with respect to topodetic coordinates as shown in the figure to the right. In this figure, A also stands for antenna and V means vehicle being tracked. The location of the vehicle in TOPodetic coordinates of EAST, NORTH, UP is denoted by $X_{V/A,TOP}$, $Y_{V/A,TOP}$ and $Z_{V/A,TOP}$.



Measured azimuth angle is given by

$$A^* = \arctan \frac{X_{V/A,TOP}}{Y_{V/A,TOP}} + B_A + q_A$$

where

$$0 \le A^* < 360°$$

B_A = the azimuth angle bias error, an exponentially correlated random variable, due to hardware errors and unmodeled refraction effects.

q_A = the uncorrelated error adding to the measurement.

The nonzero partial derivatives in topodetic coordinates are given by

$$\frac{\partial A^*}{\partial X_{V/A,TOP}} = \frac{Y_{V/A,TOP}}{X^2_{V/A,TOP} + Y^2_{V/A,TOP}}$$

$$\frac{\partial A^*}{\partial Y_{V/A,TOP}} = -\frac{X_{V/A,TOP}}{X^2_{V/A,TOP} + Y^2_{V/A,TOP}}$$

and

$$\frac{\partial A^*}{\partial B_A} = 1$$

The topodetic partial derivatives are converted to derivatives with respect to $\underline{R}_{V,EF}$ as shown in Section 5.1

Measured elevation angle is given by

$$E^* = \arctan \frac{Z_{V/A,TOP}}{\sqrt{X^2_{V/A,TOP} + Y^2_{V/A,TOP}}} + \Delta E + B_E + q_E$$

where E* can have values of

$$0 \leq E^* < 360°$$

Note that E* is not negative for angles below the horizon. Also E* will not have values greater than 90° except when tracking below the horizon (E = 359.5° for the radar on a hill.) The value ΔE is the refraction correction shown in Section 5.1. B_E is the bias, an exponentially correlated random variable, due to hardware errors and errors in calculating ΔE, which may be substantial. q_E is the uncorrelated error adding to the measurement.

The partial derivatives of E* in topodetic coordinates (the ΔE term is neglected) are given by

$$\frac{\partial E^*}{\partial X_{V/A,TOP}} = \frac{-Z_{V/A,TOP}\, X_{V/A,TOP}}{\rho^2 \sqrt{X^2_{V/A,TOP} + Y^2_{V/A,TOP}}}$$

$$\frac{\partial E^*}{\partial Y_{V/A,TOP}} = \frac{-Z_{V/A,TOP}\, Y_{V/A,TOP}}{\rho^2 \sqrt{X^2_{V/A,TOP} + Y^2_{V/A,TOP}}}$$

$$\frac{\partial E^*}{\partial Z_{V/A,TOP}} = \frac{1}{\rho^2} \sqrt{X^2_{V/A,TOP} + Y^2_{V/A,TOP}}$$

and

$$\frac{\partial E^*}{\partial B_E} = 1$$

The topodetic partial derivatives are converted to derivatives with respect to $\underline{R}_{V,EF}$ as shown in Section 5.1.

For both the azimuth and elevation angle measurements, typical skin track statistics are given by

$$\tau_B = 108 \text{ seconds}$$

For a TPQ-18 or FPQ-16 C-band radar

$$\sigma_B = 0.3 \text{ milliradians}$$

$$\sigma_q = 0.15 \text{ milliradians}$$

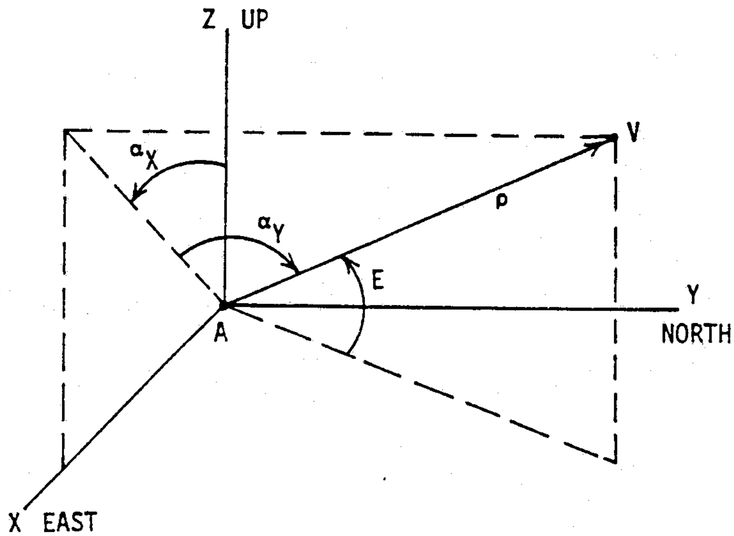
For an FPS-16 C-band radar

$$\sigma_B = 0.4 \text{ milliradians}$$

$$\sigma_q = 0.2 \text{ milliradians}$$

5.4 NORTH-SOUTH KEYHOLE ANGLE MEASUREMENT

The α_X amd α_Y angle measurements are generally made with a 9 meter (30 feet) S-band antenna. The antenna is labeled A and the tracked vehicle V. The location of the vehicle in TOPodetic coordinates of EAST, NORTH, UP is denoted by $X_{V/A,TOP}$, $Y_{V/A,TOP}$, and $Z_{V/A,TOP}$.



Measured α_X is given by

$$\alpha_X^* = \arctan \frac{X_{V/A,TOP}}{Z_{V/A,TOP}} + \Delta\alpha_X + B_{\alpha X} + q_X$$

where α_X^* can have values of *

$$-88° \le \alpha_X \le 88°$$

$\Delta\alpha_X$ is the atmospheric refraction correction. $B_{\alpha X}$ is the bias, an exponentially correlated random variable, due to hardware errors and errors in $\Delta\alpha_X$. q_X is the uncorrelated error adding to the measurements

Let ΔE be the refraction correction term for E as shown in Section 5.1. Then $\Delta\alpha_X$ is given by

*Actually $\alpha_X^* = -10°$ will read out as 350°, etc.

$$\Delta\alpha_X = \frac{-\rho^2 X_{V/A,TOP}\,\Delta E}{\sqrt{X^2_{V/A,TOP} + Y^2_{V/A,TOP}}\,(X^2_{V/A,TOP} + Z^2_{V/A,TOP})}$$

Neglecting $\Delta\alpha_X$, the topodetic partial derivatives of α^*_X are given by

$$\frac{\partial\alpha^*_X}{\partial X_{V/A,TOP}} = \frac{Z_{V/A,TOP}}{X^2_{V/A,TOP} + Z^2_{V/A,TOP}}$$

$$\frac{\partial\alpha^*_X}{\partial Y_{V/A,TOP}} = 0$$

$$\frac{\partial\alpha_X}{\partial Z_{V/A,TOP}} = -\frac{X_{V/A,TOP}}{X^2_{V/A,TOP} + Z^2_{V/A,TOP}}$$

$$\frac{\partial\alpha_X}{\partial B_{\alpha X}} = 1$$

The topodetic partial derivatives are converted to derivatives with respect to $\underline{R}_{V,EF}$ as shown in Section 5.1.

Measured α_Y is given by

$$\alpha^*_Y = \arctan\frac{Y_{V/A,TOP}}{\sqrt{X^2_{V/A,TOP} + Z^2_{V/A,TOP}}} + \Delta\alpha_Y + B_{\alpha Y} + q_Y$$

where α_Y^* is hardware limited to *

$$-80° \le \alpha_Y^* \le 80°$$

which causes a "keyhole" to exist along the north-south axis.

$\Delta\alpha_Y$ is the atmospheric refraction correction.

$B_{\alpha Y}$ is the bias, an exponentially correlated random variable, due to hardware errors and errors in $\Delta\alpha_Y$.

q_Y is the uncorrelated error adding to the measurement.

Let ΔE be the refraction correction term for the elevation angle, E, as shown in Section 5.1. Then $\Delta\alpha_Y$ is given by

$$\Delta\alpha_Y = \frac{-Y_{V/A,TOP}\, Z_{V/A,TOP}\, \Delta E}{\sqrt{X^2_{V/A,TOP} + Y^2_{V/A,TOP}}\ \sqrt{X^2_{V/A,TOP} + Z^2_{V/A,TOP}}}$$

Neglecting $\Delta\alpha_Y$, the topodetic partial derivatives of α_Y^* are given by

$$\frac{\partial\alpha_Y^*}{\partial X_{V/A,TOP}} = -\frac{Y_{V/A,TOP}\, X_{V/A,TOP}}{\rho^2\sqrt{X^2_{V/A,TOP} + Z^2_{V/A,TOP}}}$$

$$\frac{\partial\alpha_Y^*}{\partial Y_{V/A,TOP}} = \frac{1}{\rho^2}\sqrt{X^2_{V/A,TOP} + Z^2_{V/A,TOP}}$$

---

*Again $\alpha_Y^* = -10°$ will read out as 350°, etc.

$$\frac{\partial \alpha_Y^*}{\partial Z_{V/A,TOP}} = - \frac{Y_{V/A,TOP}\, Z_{V/A,TOP}}{\rho^2 \sqrt{X^2_{V/A,TOP} + Z^2_{V/A,TOP}}}$$

and

$$\frac{\partial \alpha_Y^*}{\partial B_{\alpha Y}} = 1$$

The topodetic partial derivatives are converted to derivatives with respect to $\underline{R}_{V,EF}$ as shown in Section 5.1.

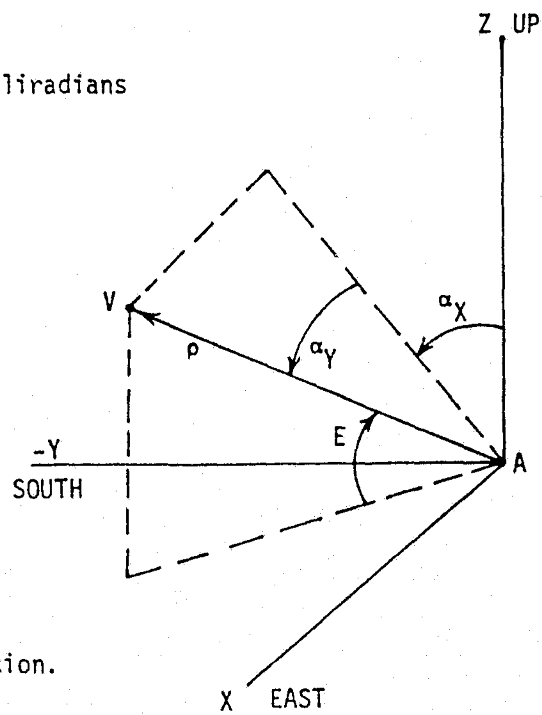
Typical error statistics for both α_X and α_Y are given below. For the angle bias time constant we have τ_B = 108 seconds.

$$\sigma_{B\alpha X} = \sigma_{B\alpha Y} = 1.6 \text{ milliradians*}$$

$$\sigma_{qX} = \sigma_{qY} = 0.4 \text{ milliradians}$$

5.5 EAST-WEST KEYHOLE ANGLE MEASUREMENTS

The α_X and α_Y angle measurements are generally made with an 85 foot S-band antenna. The antenna is labeled A and the tracked vehicle V. The location of the vehicle in TOPodetic coordinates of EAST NORTH, UP is denoted by $X_{V/A,TOP}$, $Y_{V/A,TOP}$ and $Z_{V/A,TOP}$.



*0.4 milliradians for a well calibrated station.

Measured α_X is given by

$$\alpha_X^* = \arctan \frac{-Y_{V/A,TOP}}{Z_{V/A,TOP}} + \Delta\alpha_X + B_{\alpha X} + q_X$$

where α_X^* is hardware limited to a range of *

$$-88° \leq \alpha_X \leq 88°$$

$\Delta\alpha_X$ is the atmospheric refraction correction term

$B_{\alpha X}$ is the bias, an exponentially correlated random variable, due to hardware errors and errors in $\Delta\alpha_X$.

q_X is the uncorrelated error adding to the measurement.

Let ΔE be the refraction correction term for E as shown in Section 5.1. Then $\Delta\alpha_X$ is given by

$$\Delta\alpha_X = \frac{\rho^2 \, Y_{V/A,TOP} \, \Delta E}{\sqrt{X^2_{V/A,TOP} + Y^2_{V/A,TOP}} \; (Y^2_{V/A,TOP} + Z^2_{V/A,TOP})}$$

Neglecting $\Delta\alpha_X$, the nonzero topodetic partial derivatives of α_X^* are given by

*A value of α_X = - 10° would actually read out 350°, etc.

$$\frac{\partial \alpha_X^*}{\partial Y_{V/A,TOP}} = - \frac{Z_{V/A,TOP}}{Y^2_{V/A,TOP} + Z^2_{V/A,TOP}}$$

$$\frac{\partial \alpha_X^*}{\partial Z_{V/A,TOP}} = \frac{Y_{V/A,TOP}}{Y^2_{V/A,TOP} + Z^2_{V/A,TOP}}$$

and

$$\frac{\partial \alpha_X^*}{\partial B_{\alpha X}} = 1$$

The topodetic partial derivatives are converted to derivatives with respect to $\underline{R}_{V,EF}$ as shown in Section 5.1.

Measured α_Y is given by

$$\alpha_Y^* = \arctan \frac{X_{V/A,TOP}}{\sqrt{Y^2_{V/A,TOP} + Z^2_{V/A,TOP}}} + \Delta\alpha_Y + B_{\alpha Y} + q_Y$$

α_Y^* is hardware limited to *

$$-80° \le \alpha_Y^* \le 80°$$

which causes a "keyhole" to exist along the east-west axis.

*A value of $\alpha_Y^* = -10°$ actually reads out 350°, etc.

$\Delta\alpha_Y$ is the atmospheric refraction correction.

$B_{\alpha Y}$ is the bias, an exponentially correlated random variable, due to hardware errors and errors in $\Delta\alpha_Y$.

q_Y is the uncorrelated error adding to the measurement.

Let ΔE be the refraction correction term for the elevation angle, E, as shown in Section 5.1. Then $\Delta\alpha_Y$ is given by

$$\Delta\alpha_Y = \frac{- X_{V/A,TOP}\, Z_{V/A,TOP}\, \Delta E}{\sqrt{X^2_{V/A,TOP} + Y^2_{V/A,TOP}}\ \sqrt{Y^2_{V/A,TOP} + Z^2_{V/A,TOP}}}$$

Neglecting $\Delta\alpha_Y$, the topodetic partial derivatives of α^*_Y are

$$\frac{\partial\alpha^*_Y}{\partial X_{V/A,TOP}} = \frac{1}{\rho^2} \sqrt{Y^2_{V/A,TOP} + Z^2_{V/A,TOP}}$$

$$\frac{\partial\alpha^*_Y}{\partial Y_{V/A,TOP}} = - \frac{X_{V/A,TOP}\, Y_{V/A,TOP}}{\rho^2 \sqrt{Y^2_{V/A,TOP,} + Z^2_{V/A,TOP}}}$$

$$\frac{\partial\alpha^*_Y}{\partial Z_{V/A,TOP}} = - \frac{X_{V/A,TOP}\, Z_{V/A,TOP}}{\rho^2 \sqrt{Y^2_{V/A,TOP} + Z^2_{V/A,TOP}}}$$

and

$$\frac{\partial\alpha^*_Y}{\partial B_{\alpha Y}} = 1$$

The topodetic partial derivatives are converted to derivatives with respect to the elements of the state vector, $\underline{R}_{V,EF}$, as shown in Section 5.1.

Typical error statistics for α_X and α_Y are given below. For the angle bias time constant we have τ_B = 108 seconds.

$$\sigma_{B\alpha X} = \sigma_{B\alpha Y} = 1.6 \text{ milliradians}$$

$$\sigma_{q_X} = \sigma_{q_Y} = 0.4 \text{ milliradians}$$

5.6 INTEGRATED TWO-WAY DOPPLER MEASUREMENTS

Integrated two-way Doppler measurements at an S-band station are obtained in the following manner. A signal of frequency $f_{tr} \approx 2 \cdot 10^9$ cycles/second is transmitted from an earth based antenna, A, and received at the vehicle, V, as a signal with shifted frequency, f_V. This signal is instantaneously retransmitted with a frequency of $k_V f_V$, and received back at the ground station with a frequency of f_{ob}. The frequency of this observed signal is modified by the equation

$$F = f_b + k_S(f_{ob} - k_V f_{tr})$$

in order to increase the precision of the integration procedure. Multiplication by the factor k_S increases the precision of the integration procedure, and f_b adds a bias which insures that F never goes negative.

For the Apollo missions k_V = 240/221, k_S =-1, and $f_b = 10^6$ cycles/second. The station equipment has been modified for the Space Shuttle missions. For The shuttle missions k_V = 240/221, k_S = 1000, and the biasing frequency is $240 \cdot 10^6$ cycles/second. There will be two transmitted frequencies, f_{tr}, for the shuttle. Both frequencies will be about $f_{tr} \approx 2 \cdot 10^9$ cycles/second.

The basic measurement made at the S-band station is a cycle count, N, the integral of the composite frequency, F. It can be shown (ref. 1) that the actual cycle count at the observation time T_o is

$$N^* = f_b(T_0 - T_{0I}) - 2\frac{k_S k_V f_{tr}}{c}(\rho + \Delta\rho - \frac{1}{c}\rho\dot{\rho}) + I_D + q_N$$

where

c = speed of light in a vacuum, 299 792 500 meters/second.

ρ, $\dot{\rho}$ = range and range rate at $T = T_0$.

$\Delta\rho$ = range refraction correction

T_{0I} = initialization time. Time at which the Doppler integration constant is initialized.

q_N = uncorrelated error adding to the Doppler measurement. $\sigma_{qN} \approx 120$ cycles = 8 millimeters.

I_D = Doppler constant of integration plus the integral of the rate bias error, $\sigma_\omega n_\omega$.

$$I_D = I + \int_{T_{0I}}^{T_0} \sigma_\omega n_\omega dt$$

$$I_{D,i} = I_{D,i-1} + \sigma_\omega n_{\omega,i-1}\Delta T$$

where n_ω has a unit variance and σ_ω is the variance of the rate bias error (see Section 4). I_D is initialized with values of ρ and N^* existing at $T = T_{0I}$,

$$I_D = N^* (T_{0I}) + 2 \frac{k_S k_V f_{tr}}{c} \rho (T_{0I})$$

Note that the small terms $\Delta\rho$ and $\rho\dot{\rho}/c$ are ignored in the initialization and I_D is given a large initial variance,

$$\sigma_{ID}^2 = \left(2 \frac{k_S k_V f_{tr}}{c}\right)^2 (\sigma_X^2 + \sigma_Y^2 + \sigma_Z^2)$$

The nonzero partial derivatives of N^* with respect to elements of the state vector are ($\Delta\rho$ and $\rho\dot{\rho}/c$ are ignored).

$$\frac{\partial N^*}{\partial X_{V,EF}} = -2 \frac{k_S k_V f_{tr}}{c} \frac{X_{V/A,EF}}{\rho}$$

$$\frac{\partial N^*}{\partial Y_{V,EF}} = -2 \frac{k_S k_V f_{tr}}{c} \frac{Y_{V/A,EF}}{\rho}$$

$$\frac{\partial N^*}{\partial Z_{V,EF}} = -2 \frac{k_S k_V f_{tr}}{c} \frac{Z_{V/A,EF}}{\rho}$$

$$\frac{\partial N^*}{\partial I_D} = 1$$

6.0 THE CONVERSION FROM GEODETIC COORDINATES TO EARTH-FIXED COORDINATES

The station locations are input to the TRW Bench Program in terms of the geodetic coordinates:

ϕ = geodetic latitude

λ = east longitude

h = altitude above the reference ellipsoid

Let

R_E = equatorial radius of the reference ellipsoid.

R_P = polar radius of the reference ellipsoid.

f = $(R_E - R_P)/R_E$ = flattening or ellipticity of the reference ellipsoid.

Typical values are R_E = 6 378 166 meters and f = 1/298.3.

The cartesian, earth-fixed coordinates are X_{EF}, Y_{EF}, Z_{EF} where X_{EF} and Y_{EF} are in the earth's equatorial plane (normal to the mean axis of rotation) and Z_{EF} lies along the earth's mean axis of rotation. X_{EF} lies in the plane formed by the Greenwich meridian and passes through 0° latitude, 0° longitude. Y_{EF} passes through 0° latitude and 90° east longitude.

The earth-fixed, EF, coordinates are given by

$$X_{EF} = \left[\frac{R_E}{\sqrt{\cos^2 \phi + (1 - f)^2 \sin^2 \phi}} + h \right] \cos \phi \cos \lambda$$

$$Y_{EF} = \left[\frac{R_E}{\sqrt{\cos^2 \phi + (1 - f)^2 \sin^2 \phi}} + h \right] \cos \phi \sin \lambda$$

$$Z_{EF} = \left[\frac{R_E (1 - f)^2}{\sqrt{\cos^2 \phi + (1 - f)^2 \sin^2 \phi}} + h \right] \sin \phi$$

Though not used by the program, the inverse problem is of interest. To find ϕ, λ, h given X_{EF}, Y_{EF}, Z_{EF} is a more difficult problem. λ is easily obtained from

$$\lambda = \arctan (Y_{EF}/X_{EF})$$

Let $R_{XY} = \sqrt{X_{EF}^2 + Y_{EF}^2}$. Then the task of determining ϕ and h is simplified if we first determine B by iterating four or five times

$$B = \frac{f(2 - f)\, R_E}{\sqrt{R_{XY}^2/(B + 1)^2 + (1 - f)^2\, Z_{EF}^2}}$$

using a starting value for B of .0067. Then the geodetic latitude, ϕ, is given by

$$\phi = \arctan \frac{Z_{EF}}{R_{XY}/(B + 1)}$$

Altitude, h, is given by

$$h = \left[1 - B \frac{(1 - f)^2}{f(2 - f)} \right] \sqrt{R_{XY}^2/(B + 1)^2 + Z_{EF}^2}$$

Note that for $h = 0$, $B = 1/(1 - f)^2 - 1$. For $h = \infty$, $B = 0$.

7.0 CONVERSION FROM ECI COORDINATES TO EARTH-FIXED COORDINATES

The onboard telemetry vector comes into the TRW Bench Program in earth-centered-inertial (ECI) coordinates. It is necessary to convert this vector to earth-fixed (EF) coordinates.

An ECI coordinate system has its origin at the center of the earth and is nonrotating with respect to the fixed stars. In a true inertial coordinate system there must be no rotation and a zero acceleration of the origin. Our ECI coordinate system will not be a true inertial coordinate system since its origin will experience a small acceleration as the earth moves around the sun in the solar system. However, since all bodies in the vicinity of the earth experience this same gravitational acceleration of the sun (and moon) the effects of the acceleration tend to cancel when the motion of the body is expressed in ECI coordinates.

The tranformation matrix (RNP) relates the Earth-fixed coordinated to the ECI coordinates by the following equation at time: $T = T_0$

$$\underline{R}_{ECI} = (RNP)^T \, \underline{R}_{EF}$$

Since the EF coordinate system rotates around its Z axis, the Z-rotation matrix is introduced to give R_{ECI} at time T by

$$\theta = \omega_E \, (T - T_0)$$

$$\underline{R}_{ECI} = (RNP)^T \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \underline{R}_{EF}$$

Where ω_E is the earth's angular velocity in inertial space, $\omega_E = .729211514646 \cdot 10^{-4}$ radians/second. Due to precesion and nutation of the earth's axis of rotation, RNP was changed slightly every six hours during the Apollo missions.

The above coordinate transformation matrices are orthogonal, the inverse equals the transpose, so

$$R_{EF} = \begin{bmatrix} \cos\Theta & \sin\Theta & 0 \\ -\sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} (RNP)\ \underline{R}_{ECI}$$

And, taking the derivative with respect to time yields

$$\underline{\dot{R}}_{EF} = \begin{bmatrix} \cos\Theta & \sin\Theta & 0 \\ -\sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} (RNP)\ \underline{\dot{R}}_{ECI}$$

$$+ \omega_E \begin{bmatrix} -\sin\Theta & \cos\Theta & 0 \\ -\cos\Theta & -\sin\Theta & 0 \\ 0 & 0 & 0 \end{bmatrix} (RNP)\ \underline{R}_{ECI}$$

But

$$(RNP)\ R_{ECI} = \begin{bmatrix} \cos\Theta & -\sin\Theta & 0 \\ \sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \underline{R}_{EF}$$

So

$$\dot{R}_{EF} = \begin{bmatrix} \cos\Theta & \sin\Theta & 0 \\ -\sin\Theta & \cos\Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} (RNP)\ \dot{R}_{ECI}$$

$$+ \omega_E \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \underline{R}_{EF}$$

8.0 THE KALMAN FILTER EQUATIONS

The Kalman filter equations used by the program are shown below in Table 1.

1)	$\emptyset = \partial f / \partial \underline{x}$	form transition matrix
2)	$\underline{x} = f(\underline{x},\ t_{n-1},\ \Delta T)$	integrate state vector
3)	$C = \emptyset C\ \emptyset^T + S$	propagate error covariance matrix†
	DO 14 I = 1, M	M = number of meas. at time t_n
4)	$\hat{y}_I = g_I\ (\underline{x}_n,\ t_n)$	estimate Ith measurement
5)	$P = \partial g_I / \partial \underline{x}$	form measurement partials
6)	$A = CP^T$	temporary storage vector
7)	CALCULATE NONLINEAR CORRECTION TERM ΔQ.	
8)	$R = PA + Q_I + \Delta Q$	estimate residual variance
9)	LOAD SCALAR MEASUREMENT, y_I^*, FOR TIME t_n.	
10)	IF $(y_I^* - \hat{y}_I)^2 > 36R$ GO TO 14	6σ residual edit
11)	$W = A/R$	form residual weighting vector
12)	$\underline{x} = \underline{x} + W(y_I^* - \hat{y}_I)$	correct state vector
13)	$C = C - WA^T$	correct error covariance matrix†
14)	CONTINUE	
	Gϕ Tϕ 1	

†Make use of symmetry in steps 3 and 13.

Table 1. The Kalman Filter Equations

Making use of symmetry in steps 3 and 13 is essential for trouble free performance of the Kalman filter. Calculate only the lower triangular part of C, and set the upper triangular part equal to the lower triangular part. This is, of course, faster but most importantly it forces C to be a symmetric matrix. Theoretically the equations in Table 1 give a symmetric C matrix. However, due to computer programming, the sequence of operations for the upper and lower parts C are different, and roundoff error can cause C to become increasingly nonsymmetric. When this happens, the filter may become unstable and blow up. Generally when this occurs, the investigator will find negative variances along the diagonal of C. He then may mistakenly assume that, since the error covariance is not positive semidefinite, this is the reason that the filter blew up.

When the estimated measurement $\hat{y} = g(\underline{x}, t_n)$ is a highly nonlinear function of $\underline{x}$, and when the error in the estimate of $\underline{x}$ is large, then there may be severe difficulty in getting the Kalman filter to converge. What is apparent to the investigator is that the error covariance matrix will decrease to values that are much smaller than the actual errors in the state vector. The filter will tend to ignore the current measurements even though the residual may be large. In these cases a nonlinear correction term, ΔQ, should be calculated. It can be shown that ΔQ is given by the following equations (ref. 2).

Let

$$g_{ij} = \frac{\partial g}{\partial x_i \, \partial x_j} \qquad \text{(measurement partials)}$$

$$C^{ij} = \text{The ij element of C}$$

Then

$$\Delta Q = \frac{1}{2} g_{K\ell} \, C^{km} \, C^{\ell n} \, g_{mn} + \left(\frac{1}{2} g_{K\ell} \, C^{K\ell}\right)^2$$

where the repeated subscripts and superscripts mean a summation from 1 to N, the number of elements in the state vector.

The above value of ΔQ has been found to work very well in the Space Shuttle onboard navigation equations. The use of ΔQ in effect increases the value of the measurement noise variance, Q. This causes the error covariance matrix, C, to decrease more slowly than normal, and the corrections to the state vector are not as large as with no ΔQ. Since the measurement residuals are weighted less using ΔQ, it is referred to as measurement underweighting. The value of ΔQ generally becomes small after the first few measurements are processed and can generally (but not always) be ignored thereafter.

We note that the above equation for ΔQ is quite complicated. In reference 2 an alternate, simpler method for calculating ΔQ is given which we will use in the High Speed Tracking Data Processor. For all the measurements except the Doppler measurement, when the position error is large ΔQ is calculated by

$$\Delta Q = 0.2\ PCP^T = 0.2\ PA$$

The residual variance is then given by

$$R = 1.2\ PA + Q$$

where 1.2 is the measurement underweighting factor. This factor is set to 1 when the position error variance, C(1,1) + C(2,2) + C(3,3), is less than (1000 meters)^2. For the Doppler measurement, an underweighting factor of 3 all the time was found to be necessary. $\Delta Q = 2\ PCP^T = 2\ PA$ in this case.

9.0 TRW BENCH PROGRAM FLOW DIAGRAM

The overall flow diagram for the TRW Bench Program High-Speed-Tracking-Data Processor is shown in Figure 1. Each block in the figure (there are seven blocks) is a subroutine called by the main program. The PS(I) and P(I) arrays mentioned in the figure are input and calculated constants. The NF(I) array consists of logic flags. The equation

$$I = (T_{REAL} - T_{LAG})/\Delta T$$

means that I is set to the next lowest integer, just as would be done when using FORTRAN. In block 6, PBI stands for push button indicator.

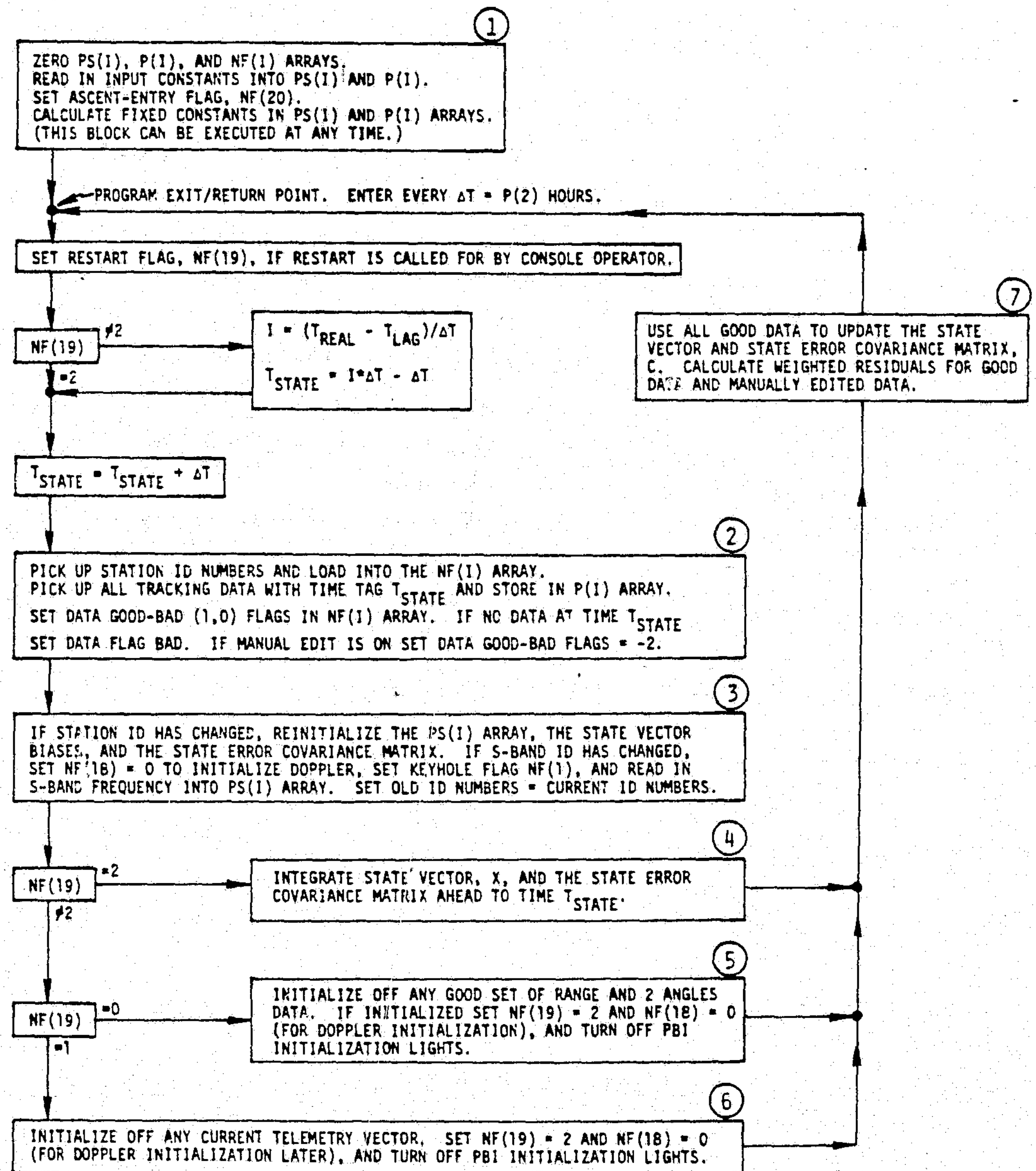


FIGURE 1: FLOW DIAGRAM FOR THE HIGH-SPEED-TRACKING-DATA PROCESSOR

10.0 TRW BENCH PROGRAM INPUTS AND OTHER CONSTANTS

The PS(I) and P(I) arrays are used for nondestroyable storage in the program. There are 72 elements in each array. The PS(I) array is used for tracking station related parameters. The definitions of each element in the arrays are shown in Sections 10.1 and 10.2. Where suitable, typical values of the elements of the array will be shown. The units used will be earth radii (6 378 165 meters), hours, radians, and cycles. (Cycles are used for the Doppler measurement.) These are the units used with the actual real-time program. However, the program shown in this report may use any system of units for length and time, so long as one is consistent throughout the arrays. For example, all preliminary design work was done using meters, seconds, radians, and cycles. Note that all values used in the arrays are zeroed at the beginning of block 1.

Asterisks will denote the type of constant:

*   premission program input,

** real time program input,

no asterisk, parameter determined by program.

10.1 THE PS(I) ARRAY

The station characteristics constants for the number 1 C-band station are stored in PS(I) I = 1, 20. The station characteristics constants for the number 2 C-band station are stored in PS(I) I = 21, 40. The station characteristics constants for the S-band station are stored in PS(I) I = 41, 67.

**PS(1) = h altitude of the first C-band station above the reference ellipsoid.† Typical value is $1.2480 \cdot 10^{-4}$ er (796.0 meters).

†The reference ellipsoid is specified by the constants P(17) and P(18).

**PS(2) = ϕ, geodetic latitude of the first C-band station with respect to the the reference ellipsoid. Typical value would be .61018385 radians.

**PS(3) = λ, east longitude of first C-band station, 4.22524581 radians.

**PS(4) = $N_o = n_o - 1$, refraction modulus for the first C-band station. The index of refraction minus 1. This number changes from month to month. A typical value would be .0003307, no units.

**PS(5) = H_S, the atmospheric scale height used in the refraction corrections at the first C-band station. Literally the height of a spherical slab atmosphere whose refraction modulus is everywhere N_o. H_S changes from month to month. Typical value = .00104 er (6633 meters).

**PS(6) = σ for the range bias at the first C-band station, $2.8 \cdot 10^{-6}$ er (18 meters $\approx$ 60 feet).

**PS(7) = σ for azimuth bias at first C-band station, .0004 radians (0.4 milliradians).

**PS(8) = σ for elevation angle bias at first C-band station, .0004 radians (0.4 milliradians).

**PS(9) = σ for uncorrelated error adding to the range measurement at the number 1 C-band station, $1.4 \cdot 10^{-6}$ er (9 meters).

**PS(10) = σ for the uncorrelated error adding to the azimuth measurement at the number 1 C-band station, .0002 radians (0.2 milliradians).

**PS(11) = σ for the uncorrelated error adding to the elevation angle measurement at the number 1 C-band station, .0002 radians (0.2 milliradians).

PS(12) $= \sigma_\rho^2 \, [1 - \exp(-2\Delta T/\tau_B)]$, number 1 C-band state noise variance for the range bias.

PS(13) $= \sigma_A^2 \, [1 - \exp(-2\Delta T/\tau_B)]$, number 1 C-band state noise variance for the azimuth bias.

$PS(14) = \sigma_E^2\,[1 - \exp(-2\Delta T/\tau_B)]$, number 1 C-band state noise variance for the elevation angle bias.

PS(15) = PS(9)**2, variance of the uncorrelated error adding to the range measurement for the number 1 C-band station.

PS(16) = PS(10)**2, variance of the uncorrelated error adding to the azimuth measurement for the number 1 C-band station.

PS(17) = PS(11)**2, variance of the uncorrelated error adding to the elevation angle measurement for the number 1 C-band station.

$$\left.\begin{aligned} PS(18) &= X_{EF} \\ PS(19) &= Y_{EF} \\ PS(20) &= Z_{EF} \end{aligned}\right\}\ \text{Earth-fixed coordinates of the first C-band station.}$$

The station characteristics constants for the second C-band station are shown below.

**PS(21) = h, altitude of second C-band station above the reference ellipsoid. Typical value is $1.568\cdot10^{-5}$ er (100 meters).

**PS(22) = ϕ, geodetic latitude of second C-band station with respect to the reference ellipsoid. Typical value is .60503594 radians.

**PS(23) = λ, east longitude of second C-band station, 4.17864945 radians

**PS(24) = $N_o = n_o - 1$, modulus of refraction for the second C-band station. The index of refraction minus 1. This number changes from month to month. A typical value would be .0003307, no units.

**PS(25) = H_S, the atmospheric scale height used in the refraction corrections at the second C-band station. Literally the height of a spherical slab atmosphere whose refraction modulus is everywhere N_o. H_S changes from month to month. Typical value = .00104 er (6633 meters).

**PS(26) = σ for the range bias at the second C-band station, $1.9 \cdot 10^{-6}$ er (12 meters = 40 feet).

**PS(27) = σ for azimuth bias at the second C-band station, .0003 radians (0.3 milliradians).

**PS(28) = σ for elevation angle bias at the second C-band station, .0003 radians (0.3 milliradians).

**PS(29) = σ for uncorrelated error adding to the range measurement at the second C-band station, $.95 \cdot 10^{-6}$ er (6 meters = 20 feet).

**PS(30) = σ for uncorrelated error adding to the azimuth measurement at the second C-band station, .00015 radians (0.15 milliradians).

**PS(31) = σ for uncorrelated error adding to the elevation angle measurements at the second C-band station, .00015 radians (0.15 milliradians).

PS(32) = $\sigma_\rho^2\,[1 - \exp(-2\Delta T/\tau_B)]$, second C-band state noise variance for the range bias.

PS(33) = $\sigma_A^2\,[1 - \exp(-2\Delta T/\tau_B)]$, second C-band state noise variance for the azimuth bias.

PS(34) = $\sigma_E^2\,[1 - \exp(-2\Delta T/\tau_B)]$, second C-band state noise variance for the elevation angle bias.

PS(35) = PS(29)**2, variance of the uncorrelated error adding to the range measurement at the second C-band station.

PS(36) = PS(30)**2, variance of the uncorrelated error adding to the azimuth measurement at the second C-band station.

PS(37) = PS(31)**2, variance of the uncorrelated error adding to the elevation angle measurement at the second C-band station.

$PS(38) = X_{EF}$

$PS(39) = Y_{EF}$ } Earth-fixed coordinates of second C-band station.

$PS(40) = Z_{EF}$

The station characteristics constants for the S-band station are shown below.

**PS(41) = h, altitude of the S-band station above the reference ellipsoid. Typical value is $1.872 \cdot 10^{-4}$ er (1194 meters).

**PS(42) = ϕ, geodetic latitude of the S-band station with respect to the reference ellipsoid. Typical value is .61202125 radians.

**PS(43) = λ, east longitude of the S-band station, -2.03838859 radians.

**PS(44) = $N_o = n_o - 1$, modulus of refraction for the S-band station. The index of refraction minus 1. This number changes from month to month. A typical value would be .0003307, no units.

**PS(45) = H_S, the atmospheric scale height used in the refraction correction equations for the S-band site. Literally the height of a spherical slab atmosphere whose refraction modulus is everywhere N_o. H_S changes from month to month. Typical value = .00104 er (6633 meters).

**PS(46) = σ for the range bias at the S-band station, $2.8 \cdot 10^{-6}$ er (18 meters).

**PS(47) = σ for the α_X bias at the S-band station, .0016 radians (1.6 milliradians). For a well calibrated station σ = .0004 radians.

**PS(48) = σ for the α_Y bias at the S-band station, .0016 radians (1.6 milliradians). For a well calibrated station σ = .0004 radians.

**PS(49) = σ for the uncorrelated error adding to the range measurement at the S-band station, $2.3 \cdot 10^{-6}$ er (14.5 meters).

**PS(50) = σ for the uncorrelated error adding to the α_X measurement at the S-band station, .0004 (0.4 milliradians).

**PS(51) = σ for the uncorrelated error adding to the α_Y measurement at the S-band station, .0004 radians (0.4 milliradians).

**PS(52) = σ for the uncorrelated error adding to the Doppler cycle count measurement at the S-band station, 120 cycles.

PS(53) = $\sigma_\rho^2\ [1 - \exp(-2\Delta T/\tau_B)]$, S-band state noise variance for the range bias.

PS(54) = $\sigma_{\alpha X}^2\ [1 - \exp\ (-2\Delta T/\tau_B)]$, S-band state noise variance for the α_X bias.

PS(55) = $\sigma_{\alpha Y}^2\ [1 - \exp(-2\Delta T/\tau_B)]$, S-band state noise variance for the α_Y bias.

PS(56) = PS(49)**2, variance of the uncorrelated error adding to the range measurement at the S-band station.

PS(57) = PS(50)**2, variance of the uncorrelated error adding to the α_X measurement at the S-band station.

PS(58) = PS(51)**2, variance of the uncorrelated error adding to the α_Y measurement at the S-band station.

PS(59) = PS(52)**2, variance of the uncorrelated error adding to the Doppler cycle count measurements at the S-band station.

$$\left.\begin{array}{l} PS(60) = X_{EF} \\ PS(61) = Y_{EF} \\ PS(62) = Z_{EF} \end{array}\right\} \text{Earth-fixed coordinates of the S-band station.}$$

PS(63) = f_b, the biasing frequency for the Doppler measurement at the S-band station, $8.64 \cdot 10^{11}$ cycles/hour ($240 \cdot 10^6$ cycles/second).

*PS(64) = k_V, S-band frequency multiplication factor at the beacon on the vehicle, 240/221 = 1.085972851 no units.

*PS(65) = k_S, Doppler frequency multiplication factor at the S-band station, 1000 no units.

**PS(66) = f_{tr}, the S-band transmitter frequency <u>appearing in the incoming data stream</u>, not the station characteristics table, in units of cycles/second. f_{tr} will be converted to units of cycles/hour external to this program. A typical value for the Apollo mission was $7.566487603 \cdot 10^{12}$ cycles/hour ($2.101802112 \cdot 10^{9}$ cycles/second).

PS(67) = 2 $K_S k_V f_{tr}/c$ where c is the speed of light in a vacuum. Typical value for P(67) = $9.71258784 \cdot 10^{10}$ cycles/er = 15227.19902 cycles/meter.

*PS(68) = K in KΔT, the state noise variance for the Doppler integration constant, I_D, the 19th element of the state vector. Typical value = $3.6 \cdot 10^{7}$ cycles^2/hour = 10000 cycles^2/second.

PS(69) = KΔT, see above.

*PS(70) = τ_B, time constant for all measurement biases, typical value is .03 hours = 108 seconds.

PS(71) = $\exp(-\Delta T/\tau_B)$.

PS(72) = 1 - PS(71)**2.

10.2 THE P(I) ARRAY

The following constants and variables are not related to station parameters.

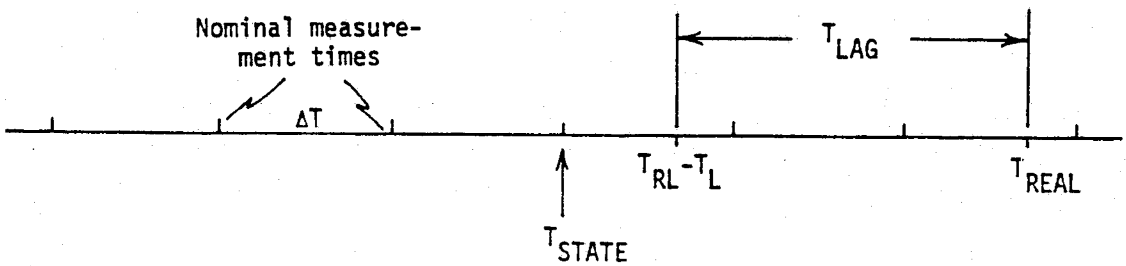
P(1) = $T_{STATE} = T_S$, time tag of state vector in hours.

*P(2) = ΔT, nominal time interval between measurements. State vector is provided every ΔT hours whether or not there are currently good measurements available. Typical value = $5.5555\ 55555\ 55556 \cdot 10^{-5}$ hours (0.2 seconds).

$P(3) = \Delta T^2/2$

*P(4) = $T_{LAG} = T_L$, filter lag time behind real time. Due to transport delay in getting data from the tracking site to the computer. Typical value = .0005 hours (1.8 seconds).

**P(5) = $T_{REAL} = T_{RL}$, real current mission time in hours, measured from the beginning of the year.



$$I = (T_{RL} - T_L)/\Delta T \qquad T_{STATE} = I * \Delta T$$

Figure 2: Determination of the Initial Filter State Time (Filter called every ΔT hours thereafter.)

*P(6) = ECRV τ_a for acceleration during ascent.† Typical value = .01111 11111 hours (40 seconds).

†ECRV stands for "exponentially correlated random variable".

*P(7) = ECRV σ_a for acceleration during ascent. Typical value is 12.2 er/hr^2 (6 m/sec^2).

*P(8) = ECRV τ_a for acceleration during entry. Typical value = .01111 11111 hours (40 seconds).

*P(9) = ECRV σ_a for acceleration during entry. Typical value is 12.2 er/hr^2 (6 m/sec^2).

P(10) = $\exp(-\Delta T/\tau_a)$ for either ascent or entry phase.

P(11) = σ_a^2 for either ascent or entry phase.

P(12) = $\sigma_a^2\ [1 - \exp(-2\Delta T/\tau_a)]$ for either ascent or entry phase.

*P(13) = position variance for initial, diagonal, state error covariance matrix for a restart from the raw tracking data. Typical value = $2.458 \cdot 10^{-6}$ er^2 (10000 meters)^2.

*P(14) = position variance for initial, diagonal, state error covariance matrix for a restart from a telemetry vector. Typical value = 1 er^2 (6 378 165 meters)^2.

*P(15) = velocity variance for initial, diagonal, state error covariance matrix, 20.4 er^2/hr^2 (8000 m/sec)^2.

*P(16) = residual edit parameter. P(16) = 6 edits 6σ residuals.

*P(17) = R_E, equatorial radius of the ellipsoidal figure of the earth. For the 1960 Fischer ellipsoid R_E = 1.000000157 er (6 378 166 meters).

*P(18) = f, ellipticity or flattening of the ellipsoidal figure of the earth. For the 1960 Fischer ellipsoid f = 1/298.3 = $3.352329869 \cdot 10^{-3}$ no units.

*P(19) = c, speed of light in a vacuum, 169210.5802 er/hr (299 792 500. m/sec).

*P(20) = ω_E, inertial angular velocity of the earth, .262516 145273 rad/hour (.729211 514646·10^{-4} rad/sec).

**P(21) = time tag of RNP matrix in hours.

**P(22)
**P(23)
**P(24)
**P(25)
**P(26)
**P(27)
**P(28)
**P(29)
**P(30)

The RNP matrix, column stored. RNP is used in the conversion of the ECI telemetry vector to earth-fixed (EF) coordinates. The time tag of this matrix must be for the time T_{STATE} = P(21).

*P(31) = RSS position error variance below which the measurement underweighting, nonlinear measurement correction, is turned off. Typical value = 2.458·10^{-3} er^2 $(1000 \text{ meters})^2$.

P(32) = counter for Doppler bias frequency calculation.

P(33) = scaled residual[†] for range from the first C-band station.
P(24) = scaled residual for azimuth from the first C-band station.
P(35) = scaled residual for elevation from the first C-band station.
P(36) = scaled residual for range from the second C-band station.
P(37) = scaled residual for azimuth from the second C-band station.
P(38) = scaled residual for elevation from the second C-band station.
P(39) = scaled residual for range from the S-band station.
P(40) = scaled residual for α_X from the S-band station.
P(41) = scaled residual for α_Y from the S-band station.
P(42) = scaled residual for cycle count from the S-band station.

[†] Residual divided by the filter's predicted standard deviation for the residual.

**P(43), **P(44), **P(45) = ρ, A, E data from the first C-band station in er, and radians. DOD EFG data is converted externally to ρ, A, E data.

**P(46), **P(47), **P(48) = ρ, A, E data from the second C-band station in er, radians, and radians. DOD EFG data is converted externally to ρ, A, E data.

**P(49), **P(50), **P(51), **P(52) = ρ, α_X, α_Y, and Doppler cycle count from the S-band station. Units are er, radians, radians, and cycle counts.

**P(I) I = 53, 58 is telemetry vector of position and velocity in M50 ECI coordinates. Time tag is not important.

P(59) = 1. or 1.2, the underweighting factor set by the program according to the predicted RSS position error. Used for all measurements except the Doppler measurement which uses an underweighting factor of 3 all the time.

P(I) I = 60, 72 not currently used.

10.3 THE 3 BY 3 T5 MATRICES

The T5 matrix converts EF coordinates to TOPodetic coordinates of east, north, up as discussed in Section 5.1.

$$\underline{R}_{TOP} = T5\ \underline{R}_{EF}$$

The T5 matrix is also used to convert the TOP partial derivates of the measurements back to EF coordinates. The T5 matrix is generated by the subroutine ST5.

T51 = coordinate transformation matrix for the first C-band station.

T52 = coordinate transformation matrix for the second C-band station.

T53 = coordinate transformation matrix for the S-band station.

11. THE NF(I) FLAGS (INTEGERS)

All NF(I) are set to zero, in Block 1 of Figure 1, when the program is first called. Asterisks identify the NF(I) in the following manner.

No asterisk means flag is computed by the program.

*premission input. (There currently are none.)

**real-time program input.

***both a real-time input and computed by the program.

**NF(1) identifies data type from the S-band station.†

NF(1) = 0 for a north-south keyhole.

NF(1) = 1 for an east-west keyhole.

**NF(2) is the ID number for the first C-band station.

NF(3) = last value of NF(2). If NF(2) changes from its previous value, then the measurement biases and the state error covariance matrix are reinitialized.

**NF(4) is the ID number for the second C-band station.

NF(5) = last value of NF(4). If NF(4) changes from its previous value, then the measurement biases and the state error covariance matrix are reinitialized.

**NF(6) is the ID number for the S-band station.

---

†There are no flags for the two data types from the two C-band stations. Instead, the modulus of refraction for the C-band station is checked as follows:

$N_o \neq 0$ indicates normal, , A, E data.

$N_o = 0$ indicates ρ, A, E data have been corrected for refraction and speed of light.

NF(7) = last value of NF(6). If NF(6) changes from its previous value, then the measurement biases and the state error covariance matrix are reinitialized, and NF(18) is set to zero so as to cause the Doppler integration constant to be reinitialized.

***NF(8) = 1, ρ from first C-band station has a good data tag.

= 0, ρ from first C-band station has a bad data tag.†

= -1, ρ from first C-band station has failed residual edit.

= -2, ρ from first C-band station is being manually edited by the control board operator. Residuals will continue to be calculated in this case.

***NF(9) = 1, A from first C-band station has a good data tag.

= 0, A from first C-band station has a bad data tag.

= -1, A from first C-band station has failed residual edit.

= -2, A from first C-band station is being manually edited by the control board operator. Residuals will continue to be calculated in this case.

***NF(10) = 1, E from first C-band station has a good data tag.

= 0, E from first C-band station has a bad data tag.

= -1, E from first C-band station has failed residual edit.

= -2, E from first C-band station is being manually edited by the control board operator. Residuals will continue to be calculated in this case.

***NF(11) = 1, ρ from second C-band station has a good data tag.

= 0, ρ from second C-band station has a bad data tag.

= -1, ρ from second C-band station has failed residual edit.

= -2, ρ from second C-band station is being manually edited by the control board operator. Residuals will continue to be calculated in this case.

---

†No data or missing data will be tagged bad.

***NF(12) = 1, A from second C-band station has a good data tag.

= 0, A from second C-band station has a bad data tag.

= -1, A from second C-band station has failed residual edit.

= -2, A from second C-band station is being manually edited by the control board operator. Residuals will continue to be calculated in this case.

***NF(13) = 1, E from second C-band station has a good data tag.

= 0, E from second C-band station has a bad data tag.

= -1, E from second C-band station has failed residual edit.

= -2, E from second C-band station is being manually edited by the control board operator. Residuals will continue to be calculated in this case.

***NF(14) = 1, ρ from S-band station has a good data tag.

= 0, ρ from S-band station has a bad data tag.

= -1, ρ from S-band station has a failed residual edit.

= -2, ρ from S-band station is being manually edited by the control board operator. Residuals will continue to be calculated in this case.

***NF(15) = 1, α_X from S-band station has a good data tag.

= 0, α_X from S-band station has a bad data tag.

= -1, α_X from S-band station has failed residual edit.

= -2, α_X from S-band station is being manually edited by the control board operator. Residuals will continue to be calculated in this case.

***NF(16) = 1, α_γ from S-band station has a good data tag.

= 0, α_γ from S-band station has a bad data tag.

= -1, α_γ from S-band station has failed residual edit.

= -2, α_γ from S-band station is being manually edited by the control board operator. Residuals will continue to be calculated in this case.

***NF(17) = 1, Doppler count from S-band station has a good data tag.

= 0, Doppler count from S-band station has a bad data tag.

= -1, Doppler count from S-band station has failed residual edit.

= -2, Doppler count from S-band station is being manually edited by the control board operator. Residuals will continue to be calculated in this case.

NF(18) = previous value of NF(17) except that NF(18) cannot equal -1. Initially set to zero by the program. IF NF(17) = 1 and NF(18) $\neq$ 1 then the Doppler integration constant is reset and the state error covariance matrix for the integration constant is reinitialized.

**NF(19) = 0, restart from tracking data. (Initially zeroed by the program).

= 1, restart from telemetry vector.

= 2, program has been initialized and is running normally. Push button indicator (PBI) is turned off.

**NF(20) = 0 for ascent.

= 1 for entry

NF(I) I = 21, 36 not currently used.

12. THE FORTRAN LISTING

12.1 THE MAIN PROGRAM

The main program inplements the flow diagram shown in Figure 1. Note that all constants and variables are placed in unlabled common storage. The double precision storage requires 1860 words. The T(19, 19) matrix is used for temporary storage. X(19) is the state vector, and C(19, 19) is the state error covariance matrix. Note that all comments in the FORTRAN listing preceded by an asterisk are programming instructions for the real-time programmer.

OP=1

```
      PROGRAM MAIN
      DOUBLE PRECISION PS,P,X,C,T,T51,T52,T53
      COMMON PS(72),P(72),X(19),C(19,19),T(19,19),T51(3,3),T52(3,3),
     A       T53(3,3),NF(36)
C     ALL COMMENTS PRECEDED BY AN ASTERISK ARE PROGRAMING INSTRUCTIONS FOR THE
C     REAL TIME PROGRAM.
      CALL BLK1
  100 CONTINUE
C    *IF A RESTART IS CALLED FOR BY THE CONSOLE OPERATOR, THEN SET NF(19)=0 TO
C    *RESTART FROM RAW TRACKING DATA, SET NF(19)=1 TO RESTART FROM A TELEMETRY
C    *VECTOR.
      IF(NF(19).EQ.2)GO TO 200
C    *FETCH REAL TIME IN HOURS AND STORE IN P(5).
      I=(P(5)-P(4))/P(2)
      P(1)=I*P(2)-P(2)
  200 P(1)=P(1)+P(2)
      CALL BLK2
      CALL BLK3
      IF(NF(19).EQ.2)CALL BLK4
      IF(NF(19).EQ.0)CALL BLK5
      IF(NF(19).EQ.1)CALL BLK6
      CALL BLK7
C    *PROGRAM EXIT/RETURN POINT. ENTER EVERY DELTA T=P(2) HOURS.
      GO TO 100
      END
```

12.2 THE BLK SUBROUTINES

Each numbered block in the Figure 1 flow diagram is a subroutine. There are seven blocks. The FORTRAN instructions for these subroutines are given here.

```
      SUBROUTINE BLK1
      DOUBLE PRECISION PS,P,X,C,T,T51,T52,T53
      COMMON PS(72),P(72),X(19),C(19,19),T(19,19),T51(3,3),T52(3,3),
     A       T53(3,3),NF(36)
C     ALL COMMENTS PRECEDED BY AN ASTERISK ARE PROGRAMING INSTRUCTIONS FOR THE
C     REAL TIME PROGRAM.
C     THIS BLOCK MAY BE EXECUTED AT ANY TIME PRIOR TO ENTERING THE KALMAN
C     FILTER LOOP.
      DO 100 I=1,72
      PS(I)=0.D0
  100 P(I)=0.D0
      DO 200 I=1,36
  200 NF(I)=0
C    *READ IN PROGRAM INPUTS TO THE PS(I) AND P(I) ARRAYS.
C    *FETCH THE RNP MATRIX FROM CORE AN COLUMN STORE IN P(I) I=22,30. FETCH ITS
C    *TIME TAG FROM CORE AND STORE IN P(21).
C    *SET ASCENT-ENTRY FLAG, NF(20), TO 1 IF IN ENTRY PHASE.
C     NOTE NF(20)=0 (SET ABOVE) INDICATES ASCENT PHASE.
C     CALCULATE FIXED CONSTANTS IN THE PS(I) AND P(I) ARRAYS.
      PS(69)=PS(68)*P(2)
      PS(71)=DEXP(-P(2)/PS(70))
      PS(72)=1.D0-PS(71)**2
      P(3)=P(2)**2/2.D0
      IF(NF(20).EQ.1)GO TO 300
      P(10)=DEXP(-P(2)/P(6))
      P(11)=P(7)**2
      GO TO 400
  300 P(10)=DEXP(-P(2)/P(8))
      P(11)=P(9)**2
  400 P(12)=P(11)*(1.D0-P(10)**2)
      RETURN
      END
```

```
      SUBROUTINE BLK2
      DOUBLE PRECISION PS,P,X,C,T,T51,T52,T53
      COMMON PS(72),P(72),X(19),C(19,19),T(19,19),T51(3,3),T52(3,3),
     A       T53(3,3),NF(36)
C      ALL COMMENTS PRECEDED BY AN ASTERISK ARE PROGRAMING INSTRUCTIONS FOR THE
C      PEAL TIME PROGRAM.
C     *FOR TRACKING DATA WITH A TIME TAG OF P(1) HOURS, LOAD STATION ID NUMBERS
C     *INTO NF(2) (NO. 1 C-BAND), NF(4) (NO. 2 C-BAND), AND NF(6) (S-BAND
C     *STATION).
C     *PICK UP ALL TRACKING DATA WITH A TIME TAG OF P(1) HOURS AND STORE IN P(I)
C     *ARRAY I=43,45 (FOR NO. 1 C-BAND), I=46,48 (FOR NO. 2 C-BAND), I=49,52 (FOR
C     *S-BAND STATION). IF NO DATA THEN LOAD NOTHING AND SET DATA GOOD-BAD (1,0)
C     *TO ZERO.SET DATA GOOD-BAD FLAGS TO ONE OR ZERO. DATA GOOD-BAD FLAGS ARE IN
C     *THE NF(I) ARRAY, I=8,10 (NO. 1 C-BAND), I=11,13 (NO. 2 C-BAND), I=14,17
C     *(S-BAND STATION).IF MANUAL EDIT IS ON FOR A STATION, THEN SET ALL THAT
C     *STATIONS GOOD-BAD FLAGS TO -2.
      RETURN
      END
```

```
      SUBROUTINE BLK3
      DOUBLE PRECISION PS,P,X,C,T,T51,T52,T53
      COMMON PS(72),P(72),X(19),C(19,19),T(19,19),T51(3,3),T52(3,3),
     A       T53(3,3),NF(36)
C     ALL COMMENTS PRECEDED BY AN ASTERISK ARE PROGRAMING INSTRUCTIONS FOR THE
C     REAL TIME PROGRAM.
      IF(NF(2).EQ.NF(3))GO TO 300
C    *FOR STATION NF(2),THE NUMBER 1 C-BAND STATION, PICK UP FROM THE STATION
C    *CHARACTERISTICS TABLE PS(I) I=1,11.
      DO 100 I=1,3
      T(I)=PS(I+5)**2
      PS(I+11)=T(I)*PS(72)
      PS(I+14)=PS(I+8)**2
      X(I+9)=0.DO
      DO 100 J=1,19
      C(I+9,J)=0.DO
  100 C(J,I+9)=0.DO
      DO 200 I=1,3
  200 C(I+9,I+9)=T(I)
      CALL SEF(PS(1),PS(2),PS(3),P(17),P(18),PS(18),PS(19),PS(20))
      CALL ST5(PS(2),PS(3),T51)
      NF(3)=NF(2)
  300 IF(NF(4).EQ.NF(5))GO TO 600
C    *FOR STATION NF(4), THE SECOND C-BAND STATION, PICK UP FROM THE STATION
C    *CHARACTERISTICS TABLE PS(I) I=21,31.
      DO 400 I=1,3
      T(I)=PS(I+25)**2
      PS(I+31)=T(I)*PS(72)
      PS(I+34)=PS(I+28)**2
      X(I+12)=0.DO
      DO 400 J=1,19
      C(I+12,J)=0.DO
  400 C(J,I+12)=0.DO
      DO 500 I=1,3
  500 C(I+12,I+12)=T(I)
      CALL SEF(PS(21),PS(22),PS(23),P(17),P(18),PS(38),PS(39),PS(40))
      CALL ST5(PS(22),PS(23),T52)
      NF(5)=NF(4)
```

```
 OP=1
      600 IF(NF(6).EQ.NF(7))RETURN
C         *FOR STATION NF(6), THE S-BAND STATION, PICK UP FROM THE STATION
C         *CHARACTERISTICS TABLE PS(I) I=41,52 AND SET THE NF(1) FLAG EQUAL TO 0 FOR
C         *A NORTH-SOUTH KEYHOLE, EQUAL TO 1 FOR AN EAST-WEST KEYHOLE.
C         *FROM THE DATA STORAGE BUFFER, PICK UP THE TRANSMITTER FREQUENCY IN
C         *CYCLES/SECOND AND STORE IN PS(66) IN UNITS OF CYCLES/HOUR.
          PS(59)=PS(52)**2
          PS(67)=2.D0*PS(65)*PS(64)*PS(66)/P(19)
C         CAUSE DOPPLER INTEGRATION CONSTANT TO BE INITIALIZED.
          NF(18)=0
          DO 700 I=1,3
          T(I)=PS(I+45)**2
          PS(I+52)=T(I)*PS(72)
          PS(I+55)=PS(I+48)**2
          X(I+15)=0.D0
          DO 700 J=1,19
          C(I+15,J)=0.D0
      700 C(J,I+15)=0.D0
          DO 800 I=1,3
      800 C(I+15,I+15)=T(I)
          CALL SEF(PS(41),PS(42),PS(43),P(17),P(18),PS(60),PS(61),PS(62))
          CALL ST5(PS(42),PS(43),T53)
          NF(7)=NF(6)
          RETURN
          END
```

OP=1

```
      SUBROUTINE BLK4
      DOUBLE PRECISION PS,P,X,C,T,T51,T52,T53
      COMMON PS(72),P(72),X(19),C(19,19),T(19,19),T51(3,3),T52(3,3),
     A            T53(3,3),NF(36)
C     THIS SUBROUTINE PROPAGATES THE STATE VECTOR AND THE STATE ERROR COVARIANCE
C     MATRIX AHEAD IN TIME DELTA T=P(2) HOURS.
      DO 100 I=1,3
      I3=I+3
      I6=I+6
      X(I)=X(I)+X(I3)*P(2)+X(I6)*P(3)
      X(I3)=X(I3)+X(I6)*P(2)
  100 X(I6)=P(10)*X(I6)
      DO 200 I=10,18
  200 X(I)=PS(71)*X(I)
      DO 300 I=1,3
      I3=I+3
      I6=I+6
      DO 300 J=1,19
      T(I,J)=C(I,J)+P(2)*C(I3,J)+P(3)*C(I6,J)
      T(I3,J)=C(I3,J)+P(2)*C(I6,J)
  300 T(I6,J)=P(10)*C(I6,J)
      DO 500 J=1,19
      DO 400 I=10,18
  400 T(I,J)=PS(71)*C(I,J)
  500 T(19,J)=C(19,J)
      DO 800 J=1,3
      J3=J+3
      J6=J+6
      DO 800 I=1,19
      IF(J.GT.I)GO TO 600
      C(I,J)=T(I,J)+T(I,J3)*P(2)+T(I,J6)*P(3)
  600 IF(J3.GT.I)GO TO 700
      C(I,J3)=T(I,J3)+T(I,J6)*P(2)
  700 IF(J6.GT.I)GO TO 800
      C(I,J6)=T(I,J6)*P(10)
  800 CONTINUE
      DO 900 J=10,18
      DO 900 I=J,19
```

```
      UP=I
  900 C(I,J)=T(I,J)*PS(71)
      DO 1000 I=1,19
      DO 1000 J=1,I
 1000 C(J,I)=C(I,J)
C     ADD STATE NOISE COVARIANCE MATRIX.
      C(7,7)=C(7,7)+P(12)
      C(8,8)=C(8,8)+P(12)
      C(9,9)=C(9,9)+P(12)
      C(10,10)=C(10,10)+PS(12)
      C(11,11)=C(11,11)+PS(13)
      C(12,12)=C(12,12)+PS(14)
      C(13,13)=C(13,13)+PS(32)
      C(14,14)=C(14,14)+PS(33)
      C(15,15)=C(15,15)+PS(34)
      C(16,16)=C(16,16)+PS(53)
      C(17,17)=C(17,17)+PS(54)
      C(18,18)=C(18,18)+PS(55)
      C(19,19)=C(19,19)+PS(69)
      RETURN
      END
```

```
      SUBROUTINE BLK5
      DOUBLE PRECISION PS,P,X,C,T,T51,T52,T53
      COMMON PS(72),P(72),X(19),C(19,19),T(19,19),T51(3,3),T52(3,3),
     A       T53(3,3),NF(36)
C     THIS SUBROUTINE INITIALIZES THE STATE VECTOR FROM THE RAW TRACKING DATA.
C     ALL COMMENTS PRECEDED BY AN ASTERISK ARE PROGRAMING INSTRUCTIONS FOR THE
C     REAL TIME PROGRAM.
      IF(NF(8)+NF(9)+NF(10).NE.3)GO TO 200
C     INITIALIZE USING FIRST C-BAND STATION.
C     CONVERT R,A,E TO TOPODETIC COORDINATES OF EAST, NORTH, UP.
      T(4)=P(43)*DCOS(P(45))
      T(1)=T(4)*DSIN(P(44))                                   EAST
      T(2)=T(4)*DCOS(P(44))                                   NORTH
      T(3)=P(43)*DSIN(P(45))                                  UP
C     TIMES T51 TRANSPOSE GIVES EF COORDINATES.
      DO 100 I=1,3
  100 X(I)=T51(1,I)*T(1)+T51(2,I)*T(2)+T51(3,I)*T(3)+PS(I+17)
      GO TO 800
  200 IF(NF(11)+NF(12)+NF(13).NE.3)GO TO 400
C     INITIALIZE USING SECOND C-BAND STATION.
      T(4)=P(46)*DCOS(P(48))
      T(1)=T(4)*DSIN(P(47))                                   EAST
      T(2)=T(4)*DCOS(P(47))                                   NORTH
      T(3)=P(46)*DSIN(P(48))                                  UP
C     TIMES T52 TRANSPOSE GIVES EF COORDINATES.
      DO 300 I=1,3
  300 X(I)=T52(1,I)*T(1)+T52(2,I)*T(2)+T52(3,I)*T(3)+PS(I+37)
      GO TO 800
  400 IF(NF(14)+NF(15)+NF(16).NE.3)RETURN
C     INITIALIZE USING S-BAND STATION.
      T(4)=P(49)*DCOS(P(51))
      IF(NF(1).EQ.1)GO TO 500
C     A NORTH-SOUTH KEYHOLE STATION.
      T(1)=T(4)*DSIN(P(50))                                   EAST
      T(2)=P(49)*DSIN(P(51))                                  NORTH
      T(3)=T(4)*DCOS(P(50))                                   UP
      GO TO 600
C     AN EAST-WEST KEYHOLE STATION.
```

```
SUBROUTINE  BLK5                                          03/01/77. 00.35.34.PAGE    2
 OP=1

  500 T(1)=P(49)*DSIN(P(51))                                            EAST
      T(2)=-T(4)*DSIN(P(50))                                            NORTH
      T(3)=T(4)*DCOS(P(50))                                             UP
C     TIMES T53 TRANSPOSE GIVES EF COORDINATES.
  600 DO 700 I=1,3
  700 X(I)=T53(1,I)*T(1)+T53(2,I)*T(2)+T53(3,I)*T(3)+PS(I+59)
  800 DO 900 I=4,19
  900 X(I)=0.D0
      DO 1000 I=1,361
 1000 C(I)=0.D0
      DO 1100 I=1,3
      C(I,I)=P(13)
      C(I+3,I+3)=P(15)
      C(I+6,I+6)=P(11)
      C(I+9,I+9)=PS(I+5)**2
      C(I+12,I+12)=PS(I+25)**2
 1100 C(I+15,I+15)=PS(I+45)**2
C    *TURN OFF PBI RESTART LIGHTS AND COMMANDS TO RESTART.
      NF(19)=2
C     CAUSE DOPPLER INTEGRATION CONSTANT TO BE INITIALIZED.
      NF(18)=0
      RETURN
      END
```

```
      SUBROUTINE BLK6
      DOUBLE PRECISION PS,P,X,C,T,T51,T52,T53
      COMMON PS(72),P(72),X(19),C(19,19),T(19,19),T51(3,3),T52(3,3),
     A       T53(3,3),NF(36)
C     THIS SUBROUTINE INITIALIZES THE STATE VECTOR FROM ANY CURRENT TELEMETRY
C     VECTOR.
C     ALL COMMENTS PRECEDED BY AN ASTERISK ARE PROGRAMING INSTRUCTIONS FOR THE
C     REAL TIME PROGRAM.
C     CONVERT M50 TELEMETRY VECTOR TO EF COORDINATES.
      T(7)=P(20)*(P(1)-P(21))
      T(8)=DCOS(T(7))
      T(9)=DSIN(T(7))
      DO 100 I=1,3
      T(I)=P(I+21)*P(53)+P(I+24)*P(54)+P(I+27)*P(55)
  100 T(I+3)=P(I+21)*P(56)+P(I+24)*P(57)+P(I+27)*P(58)
      X(1)=T(8)*T(1)+T(9)*T(2)
      X(2)=-T(9)*T(1)+T(8)*T(2)
      X(3)=T(3)
      X(4)=T(8)*T(4)+T(9)*T(5)+P(20)*X(2)
      X(5)=-T(9)*T(4)+T(8)*T(5)-P(20)*X(1)
      X(6)=T(6)
      DO 200 I=7,19
  200 X(I)=0.D0
C     INITIALIZE STATE ERROR COVARIANCE MATRIX.
      DO 300 I=1,361
  300 C(I)=0.D0
      DO 400 I=1,3
      C(I,I)=P(14)
      C(I+3,I+3)=P(15)
      C(I+6,I+6)=P(11)
      C(I+9,I+9)=PS(I+5)**2
      C(I+12,I+12)=PS(I+25)**2
  400 C(I+15,I+15)=PS(I+45)**2
C    *TURN OFF PBI RESTART LIGHTS AND COMMANDS TO RESTART.
      NF(19)=2
C     CAUSE DOPPLER INTEGRATION CONSTANT TO BE INITIALIZED.
      NF(18)=0
      RETURN
      END
```

OP=1

```
      SUBROUTINE BLK7
      DOUBLE PRECISION PS,P,X,C,T,T51,T52,T53
      COMMON PS(72),P(72),X(19),C(19,19),T(19,19),T51(3,3),T52(3,3),
     A       T53(3,3),NF(36)
C     THIS SUBROUTINE USES ALL GOOD DATA TO UPDATE THE STATE VECTOR, X, AND
C     THE STATE ERROR COVARIANCE MATRIX, C. ALSO CALC. ARE WEIGHTED RESIDUALS.
C     SET UNDERWEIGHTING FACTOR.
      P(59)=1.2D0
      IF(C(1,1)+C(2,2)+C(3,3).LT.P(31))P(59)=1.D0
      IF(NF(8).EQ.1.OR.NF(8).EQ.-2)CALL MEAS1
      IF(NF(8).EQ.1)CALL UPDATE
      IF(NF(9).EQ.1.OR.NF(9).EQ.-2)CALL MEAS2
      IF(NF(9).EQ.1)CALL UPDATE
      IF(NF(10).EQ.1.OR.NF(10).EQ.-2)CALL MEAS3
      IF(NF(10).EQ.1)CALL UPDATE
      IF(NF(11).EQ.1.OR.NF(11).EQ.-2)CALL MEAS4
      IF(NF(11).EQ.1)CALL UPDATE
      IF(NF(12).EQ.1.OR.NF(12).EQ.-2)CALL MEAS5
      IF(NF(12).EQ.1)CALL UPDATE
      IF(NF(13).EQ.1.OR.NF(13).EQ.-2)CALL MEAS6
      IF(NF(13).EQ.1)CALL UPDATE
      IF(NF(14).EQ.1.OR.NF(14).EQ.-2)CALL MEAS7
      IF(NF(14).EQ.1)CALL UPDATE
      IF(NF(15).EQ.1.OR.NF(15).EQ.-2)CALL MEAS8
      IF(NF(15).EQ.1)CALL UPDATE
      IF(NF(16).EQ.1.OR.NF(15).EQ.-2)CALL MEAS9
      IF(NF(16).EQ.1)CALL UPDATE
      IF(NF(17).EQ.1.AND.NF(18).NE.1)CALL INITD
C     NOTE THAT THIS CAUSES THE DOPPLER INTEGRATION CONSTANT TO BE INITIALIZED
C     AFTER A MANUAL EDIT IS TURNED OFF.THIS PRECAUTIONARY MEASURE IS TAKEN IN
C     CASE THE DOPPLER DATA GOES FROM BAD TO GOOD WHILE THE MANUAL EDIT IS ON.
      NF(18)=NF(17)
C     NOTE THAT NF(18) CAN NEVER EQUAL -1 (RESIDUAL EDIT). NF(18) CAN ONLY HAVE
C     VALUES OF 1 (DATA GOOD), 0 (DATA BAD), AND -2 (MANUAL EDIT).
      IF(NF(17).EQ.1.OR.NF(17).EQ.-2)CALL MEAS10
      IF(NF(17).EQ.1)CALL UPDATE
      RETURN
      END
```

12.3 THE UTILITY SUBROUTINES

There are five utility subroutines. The first is SUBROUTINE SEF (H, ALAT, ALON, RE, E, XEF, YEF, ZEF). This subroutine converts altitude, H, geodetic latitude, ALAT, and east longitude, ALON, to earth-fixed coordinates of XEF, YEF, and ZEF. The equations shown in Section 6 are used in the conversion with

ϕ = ALAT

λ = ALON

R_E = earth's equatorial radius

E = f, the flattening or ellipticity.

SUBROUTINE ST5 (ALAT, ALON, T5) generates the TOPodetic coordinate transformation matrix T5 shown in Section 5.1. Inputs to ST5 are

ALAT = ϕ = geodetic latitude, radians

ALON = λ = east longitude, radians

Subroutines REFRAC calculates the refraction corrections shown in Section 5.1. Inputs to REFRAC are

T(100) = range, ρ T(101) = sin E

T(102) = N_o, refraction modulus T(103) = H_S, scale height

The outputs are

T(104) = $\Delta\rho$, refraction correction adding to range.

T(105) = ΔE, refraction correction adding to elevation angle.

Other scratch storage used by REFRAC is

T(106) = cos E T(107) = $1.4 \cdot 10^6\ N_o$ T(108) = $2.7 \cdot 10^7\ N_o^{1.5}$

$T(109) = H$ $\quad$ $T(110) = H^*$ $\quad$ $T(111) = H^*/R_o$

$$T(112) = R_o[\sqrt{\sin^2 E + 2H^*/R_o} - \sin E]$$

$$T(113) = T(112)*[1 - 2.7 \cdot 10^7 \, N_o^{1.5} \frac{H^*}{R_o} (\cos E)^{1.4 \cdot 10^6 \, N_o}] = \Delta\rho/N_o$$

Subroutine INITD initializes the Doppler integration constant X(19) and assigns a large variance to it in the C matrix. It also zeros the Doppler bias frequency counter, P(32). Temporary storage is

$$T(100) = \text{range}, \rho$$

Subroutine UPDATE updates the state vector and state error covariance matrix using a current measurement and using the Kalman filter equations shown in Section 8. Temporary storage used by this subroutine is

$T(I + 20) = CP^T$ $\quad$ $I = 1, 19$ $\quad$ $T(15) = y^* - \hat{y}$

$T(16) = PCP^T + Q + \Delta Q$ $\quad$ $T(I + 40) = W = CP^T/T(16)$ $\quad$ $I = 1, 19$

Note that T(15), T(16) and T(I + 20) I = 1, 19 are inputs to this subroutine.

```
SUBROUTINE  SEF                                              03/01/77. 00.36.14.PAGE    1
  OP=1

      SUBROUTINE SEF(H,ALAT,ALON,RE,E,XEF,YEF,ZEF)
      DOUBLE PRECISION H,ALAT,ALON,RE,E,XEF,YEF,ZEF,T1,T2,T3,T4
C     THIS SUBROUTINE CONVERTS ALTITUDE, GEODETIC LATITUDE,AND EAST LONGITUDE TO
C     EARTH-FIXED CARTESIAN COORDINATES.
      T1=DCOS(ALAT)
      T2=(1-E)**2
      T3=T1**2+T2*DSIN(ALAT)**2
      T3=RE/DSQRT(T3)
      T4=T3+H
      XEF=T4*T1*DCOS(ALON)
      YEF=T4*T1*DSIN(ALON)
      ZEF=(T3*T2+H)*DSIN(ALAT)
      RETURN
      END
```

```
      SUBROUTINE ST5(ALAT,ALON,T5)
      DOUBLE PRECISION ALAT,ALON,T5,T1
      DIMENSION T5(3,3)
C     THIS SUBROUTINE GENERATES THE TOPODETIC COORD. TRANSFORMATION MATRIX, T5.
C     THE FIRST ROW OF T5 IS A UNIT VECTOR IN EF COORD. IN THE EAST DIRECTION.
C     THE SECOND ROW OF T5 IS A UNIT VECTOR IN EF COORD. IN THE NORTH DIRECTION.
C     THE THIRD ROW OF T5 IS A UNIT VECTOR IN EF COORD. IN THE VERT. DIRECTION.
      T5(1.1)=-DSIN(ALON)
      T5(1,2)= DCOS(ALON)
      T5(1,3)= 0.D0
      T5(2,3)= DCOS(ALAT)
      T5(3,3)= DSIN(ALAT)
      T5(2,1)=-T5(3,3)*T5(1,2)
      T5(2,2)=T5(3,3)*T5(1,1)
      T5(3,1)=T5(2,3)*T5(1,2)
      T5(3,2)=-T5(2,3)*T5(1,1)
      RFTURN
      END
```

```
      SUBROUTINE REFRAC
      DOUBLE PRECISION PS,P,X,C,T,T51,T52,T53
      COMMON PS(72),P(72),X(19),C(19,19),T(19,19),T51(3,3),T52(3,3),
     A     T53(3,3),NF(36)
C     THIS SUBROUTINE CALCULATES THE REFRACTION CORRECTION FOR RANGE, T(104),
C     AND THE REFRACTION CORRECTION FOR ELEVATION ANGLE, T(105).
      IF(T(100).LT..02D0*P(17).AND.T(101).LT..0105D0)T(101)=.0105D0
      T(106)=DSQRT(1.D0-T(101)**2)
      T(107)=1.406*T(102)
      T(108)=2.7D7*DSQRT(T(102))**3
      T(109)=P(17)**2+T(100)**2+2.D0*T(100)*P(17)*T(101)
      T(109)=DSQRT(T(109))-P(17)
      T(110)=(1.D0-DEXP(-T(109)/T(103)))*T(103)
      T(111)=T(110)/P(17)
      T(112)=P(17)*(DSQRT(DABS(T(101)**2+2.D0*T(111)))-T(101))
      T(113)=T(112)*(1.D0-T(108)*T(111)*T(106)**T(107))
      T(104)=T(102)*T(113)
      T(105)=(T(113)*T(106)/T(110))*(T(102)-T(104)/T(100))
      RETURN
      END
```

```
      SUBROUTINE INITD
      DOUBLE PRECISION PS,P,X,C,T,T51,T52,T53
      COMMON PS(72),P(72),X(19),C(19,19),T(19,19),T51(3,3),T52(3,3),
     A       T53(3,3),NF(36)
C     THIS SUBROUTINE RESETS THE INTEGRATION CONSTANT FOR THE INTEGRATED DOPPLER
C     MEASUREMENTS,AND ASSIGNS A LARGE VARIANCE TO IT IN THE C MATRIX. IT ALSO
C     ZEROS THE DOPPLER BIAS FREQUENCY COUNTER, P(32).
      T(100)=DSQRT((X(1)-PS(60))**2+(X(2)-PS(61))**2+(X(3)-PS(62))**2)
      X(19)=P(52)+PS(67)*T(100)
      DO 100 J=1,19
      C(19,J)=0.D0
  100 C(J,19)=0.D0
      C(19,19)=(C(1,1)+C(2,2)+C(3,3))*PS(67)**2
      P(32)=0.D0
      RETURN
      END
```

OP=1

```
      SUBROUTINE UPDATE
      DOUBLE PRECISION PS,P,X,C,T,T51,T52,T53
      COMMON PS(72),P(72),X(19),C(19,19),T(19,19),T51(3,3),T52(3,3),
     A              T53(3,3),NF(36)
      DO 100 I=1,19
      T(I+40)=T(I+20)/T(16)
      X(I)=X(I)+T(I+40)*T(15)
      DO 100 J=1,I
      C(I,J)=C(I,J)-T(I+40)*T(J+20)
  100 C(J,I)=C(I,J)
      RETURN
      END
```

12.4 THE MEAS SUBROUTINES

There are ten measurement subroutines, one for each measurement being processed by the program. The primary outputs of the subroutines are:

$T(15) = y^* - \hat{y}$, the residual.

$T(16) = P(59)*(PCP^T) + Q$, the predicted residual variance.

$= 3\ PCP^T + Q$ for the Doppler measurement

$= PCP^T + Q + \Delta Q$, see Kalman Filter Eqs. in Section 8.

$P(I + 32) =$ scaled residual $= T(15)/\sqrt{T(16)} \quad I = 1, 10.$

$T(I + 20) = CP^T \qquad I = 1, 19$

Other temporary storage for the measurement subroutines is:

$T(1), T(2), T(3) = \underline{R}_{V/A,EF}$

$T(4), T(5), T(6) = \underline{R}_{V/A,TOP}$

$T(7), T(8), T(9) =$ TOPodetic partial derivatives

$T(10) = \rho^2$, range^2

$T(11), T(12), T(13) =$ Earth-Fixed (EF) partial derivatives

$T(14) = \hat{y}$, the estimated measurement

$T(40) = \rho\dot{\rho}/c$, speed of light correction for range

$T(100) = \rho$ $T(101) = \sin E$ $T(102) = N_o$ $T(103) = H_S$

$T(104) = \Delta\rho$, refraction correction adding to range.

$T(105) = \Delta E$, refraction correction adding to elevation angle.

Additional scratch storage for each individual MEAS subroutine is contained in T(I) I = 60, 67 and is shown below.

Additional temporary storage for MEAS2 and MEAS5 is

$$T(60) = X^2_{V/A,TOP} + Y^2_{V/A,TOP}$$

Additional temporary storage for MEAS3 and MEAS6 is

$$T(60) = \sqrt{X^2_{V/A,TOP} + Y^2_{V/A,TOP}} \qquad T(61) = \rho^2\sqrt{X^2_{V/A,TOP} + Y^2_{V/A,TOP}}$$

Additional scratch storage for MEAS8, north-south keyhole, is

$$T(60) = X^2_{V/A,TOP} + Z^2_{V/A,TOP} \qquad T(61) = \sqrt{X^2_{V/A,TOP} + Y^2_{V/A,TOP}}$$

Additional storage for MEAS8, east-west keyhole is

$$T(62) = Y^2_{V/A,TOP} + Z^2_{V/A,TOP} \qquad T(61) = \sqrt{X^2_{V/A,TOP} + Y^2_{V/A,TOP}}$$

Additional storage for MEAS9, north-south keyhole, is

$$T(60) = \sqrt{X^2_{V/A,TOP} + Z^2_{V/A,TOP}} \qquad T(61) = \rho^2\sqrt{X^2_{V/A,TOP} + Z^2_{V/A,TOP}}$$

$$T(62) = Y_{V/A,TOP} / \sqrt{X^2_{V/A,TOP} + Y^2_{V/A,TOP}}$$

$$T(63) = Z_{V/A,TOP} / \sqrt{X^2_{V/A,TOP} + Z^2_{V/A,TOP}}$$

Additional storage for MEAS9, east-west keyhole, is

$$T(64) = \sqrt{Y^2_{V/A,TOP} + Z^2_{V/A,TOP}} \qquad T(65) = \rho^2 \sqrt{Y^2_{V/A,TOP} + Z^2_{V/A,TOP}}$$

$$T(66) = X_{V/A,TOP} / \sqrt{X^2_{V/A,TOP} + Y^2_{V/A,TOP}}$$

$$T(67) = Z_{V/A,TOP} / \sqrt{Y^2_{V/A,TOP} + Z^2_{V/A,TOP}}$$

The equations used in the MEAS subroutines may be found in Section 5. The Kalman filter equations used in the MEAS subroutines may be found in Section 8.

```
      SUBROUTINE MEAS1
      DOUBLE PRECISION PS,P,X,C,T,T51,T52,T53
      COMMON PS(72),P(72),X(19),C(19,19),T(19,19),T51(3,3),T52(3,3),
     A       T53(3,3),NF(36)
C     THIS SUBROUTINE IS FOR THE RANGE MEASUREMENT,R,FROM THE FIRST C-BAND
C     STATION.
      T(1)=X(1)-PS(18)
      T(2)=X(2)-PS(19)
      T(3)=X(3)-PS(20)
      T(100)=DSQRT(T(1)**2+T(2)**2+T(3)**2)
      T(11)=T(1)/T(100)
      T(12)=T(2)/T(100)
      T(13)=T(3)/T(100)
      T(14)=T(100)+X(10)
      IF(PS(4).LT.1.D-6)GO TO 100
      T(40)=(T(1)*X(4)+T(2)*X(5)+T(3)*X(6))/P(19)
      T(101)=T51(3,1)*T(11)+T51(3,2)*T(12)+T51(3,3)*T(13)
      T(102)=PS(4)
      T(103)=PS(5)
      CALL REFRAC
      T(14)=T(14)-T(40)+T(104)
  100 T(15)=P(43)-T(14)
      DO 200 I=1,19
  200 T(I+20)=C(I,1)*T(11)+C(I,2)*T(12)+C(I,3)*T(13)+C(I,10)
      T(16)=P(59)*(T(11)*T(21)+T(12)*T(22)+T(13)*T(23)+T(30))+PS(15)
      P(33)=T(15)/DSQRT(DABS(T(16)))
      IF(DABS(P(33)).GT.P(16))NF(8)=-1
      RETURN
      END
```

OP=1

```
      SUBROUTINE MEAS2
      DOUBLE PRECISION PS,P,X,C,T,T51,T52,T53
      COMMON PS(72),P(72),X(19),C(19,19),T(19,19),T51(3,3),T52(3,3),
     A            T53(3,3),NF(36)
C     THIS SUBROUTINE IS FOR THE AZIMUTH MEASUREMENT,A,FROM THE FIRST C-BAND
C     STATION.
      T(1)=X(1)-PS(18)
      T(2)=X(2)-PS(19)
      T(3)=X(3)-PS(20)
      T(4)=T51(1,1)*T(1)+T51(1,2)*T(2)+T51(1,3)*T(3)
      T(5)=T51(2,1)*T(1)+T51(2,2)*T(2)+T51(2,3)*T(3)
      T(6)=T51(3,1)*T(1)+T51(3,2)*T(2)+T51(3,3)*T(3)
      T(14)=DATAN2(T(4),T(5))+X(11)
      T(15)=P(44)-T(14)
      IF(T(15).GT.2.D0)T(15)=T(15)-6.283185308D0
      T(60)=T(4)**2+T(5)**2
      T(7)=T(5)/T(60)
      T(8)=-T(4)/T(60)
      T(11)=T(7)*T51(1,1)+T(8)*T51(2,1)
      T(12)=T(7)*T51(1,2)+T(8)*T51(2,2)
      T(13)=T(7)*T51(1,3)+T(8)*T51(2,3)
      DO 100 I=1,19
  100 T(I+20)=C(I,1)*T(11)+C(I,2)*T(12)+C(I,3)*T(13)+C(I,11)
      T(16)=P(59)*(T(11)*T(21)+T(12)*T(22)+T(13)*T(23)+T(31))+PS(16)
      P(34)=T(15)/DSQRT(DABS(T(16)))
      IF(DABS(P(34)).GT.P(16))NF(9)=-1
      RETURN
      END
```

OP=1

```
      SUBROUTINE MEAS3
      DOUBLE PRECISION PS,P,X,C,T,T51,T52,T53
      COMMON PS(72),P(72),X(19),C(19,19),T(19,19),T51(3,3),T52(3,3),
     A      T53(3,3),NF(36)
C     THIS SUBROUTINE IS FOR THE ELEVATION ANGLE MEASUREMENT,E,FROM THE FIRST
C     C-BAND STATION.
      T(1)=X(1)-PS(18)
      T(2)=X(2)-PS(19)
      T(3)=X(3)-PS(20)
      T(4)=T51(1,1)*T(1)+T51(1,2)*T(2)+T51(1,3)*T(3)
      T(5)=T51(2,1)*T(1)+T51(2,2)*T(2)+T51(2,3)*T(3)
      T(6)=T51(3,1)*T(1)+T51(3,2)*T(2)+T51(3,3)*T(3)
      T(10)=T(4)**2+T(5)**2+T(6)**2
      T(60)=DSQRT(T(4)**2+T(5)**2)
      T(61)=T(10)*T(60)
      T(14)=DATAN2(T(6),T(60))+X(12)
      IF(PS(4).LT.1.D-6)GO TO 100
      T(100)=DSQRT(T(10))
      T(101)=T(6)/T(100)
      T(102)=PS(4)
      T(103)=PS(5)
      CALL REFRAC
      T(14)=T(14)+T(105)
  100 T(15)=P(45)-T(14)
      IF(T(15).GT.2.D0)T(15)=T(15)-6.283185308D0
      T(7)=-(T(4)/T(61))*T(6)
      T(8)=-(T(5)/T(61))*T(6)
      T(9)=T(60)/T(10)
      T(11)=T(7)*T51(1,1)+T(8)*T51(2,1)+T(9)*T51(3,1)
      T(12)=T(7)*T51(1,2)+T(8)*T51(2,2)+T(9)*T51(3,2)
      T(13)=T(7)*T51(1,3)+T(8)*T51(2,3)+T(9)*T51(3,3)
      DO 200 I=1,19
  200 T(I+20)=C(I,1)*T(11)+C(I,2)*T(12)+C(I,3)*T(13)+C(I,12)
      T(16)=P(59)*(T(11)*T(21)+T(12)*T(22)+T(13)*T(23)+T(32))+PS(17)
      P(35)=T(15)/DSQRT(DABS(T(16)))
      IF(DABS(P(35)).GT.P(16))NF(10)=-1
      RETURN
      END
```

```
      SUBROUTINE MEAS4
      DOUBLE PRECISION PS,P,X,C,T,T51,T52,T53
      COMMON PS(72),P(72),X(19),C(19,19),T(19,19),T51(3,3),T52(3,3),
     A       T53(3,3),NF(36)
C     THIS SUBROUTINE IS FOR THE RANGE MEASUREMENT, R, FROM THE SECOND C-BAND
C     STATION.
      T(1)=X(1)-PS(38)
      T(2)=X(2)-PS(39)
      T(3)=X(3)-PS(40)
      T(100)=DSQRT(T(1)**2+T(2)**2+T(3)**2)
      T(11)=T(1)/T(100)
      T(12)=T(2)/T(100)
      T(13)=T(3)/T(100)
      T(14)=T(100)+X(13)
      IF(PS(24).LT.1.D-6)GO TO 100
      T(40)=(T(1)*X(4)+T(2)*X(5)+T(3)*X(6))/P(19)
      T(101)=T52(3,1)*T(11)+T52(3,2)*T(12)+T52(3,3)*T(13)
      T(102)=PS(24)
      T(103)=PS(25)
      CALL REFRAC
      T(14)=T(14)-T(40)+T(104)
  100 T(15)=P(46)-T(14)
      DO 200 I=1,19
  200 T(I+20)=C(I,1)*T(11)+C(I,2)*T(12)+C(I,3)*T(13)+C(I,13)
      T(16)=P(59)*(T(11)*T(21)+T(12)*T(22)+T(13)*T(23)+T(33))+PS(35)
      P(36)=T(15)/DSQRT(DABS(T(16)))
      IF(DABS(P(36)).GT.P(16))NF(11)=-1
      RETURN
      END
```

OP=1

```
      SUBROUTINE MEAS5
      DOUBLE PRECISION PS,P,X,C,T,T51,T52,T53
      COMMON PS(72),P(72),X(19),C(19,19),T(19,19),T51(3,3),T52(3,3),
     A      T53(3,3),NF(36)
C     THIS SUBROUTINE IS FOR THE AZIMUTH MEASUREMENT, A, FROM THE SECOND C-BAND
C     STATION.
      T(1)=X(1)-PS(38)
      T(2)=X(2)-PS(39)
      T(3)=X(3)-PS(40)
      T(4)=T52(1,1)*T(1)+T52(1,2)*T(2)+T52(1,3)*T(3)
      T(5)=T52(2,1)*T(1)+T52(2,2)*T(2)+T52(2,3)*T(3)
      T(6)=T52(3,1)*T(1)+T52(3,2)*T(2)+T52(3,3)*T(3)
      T(14)=DATAN2(T(4),T(5))+X(14)
      T(15)=P(47)-T(14)
      IF(T(15).GT.2.D0)T(15)=T(15)-6.28318530800
      T(60)=T(4)**2+T(5)**2
      T(7)=T(5)/T(60)
      T(8)=-T(4)/T(60)
      T(11)=T(7)*T52(1,1)+T(8)*T52(2,1)
      T(12)=T(7)*T52(1,2)+T(8)*T52(2,2)
      T(13)=T(7)*T52(1,3)+T(8)*T52(2,3)
      DO 100 I=1,19
  100 T(I+20)=C(I,1)*T(11)+C(I,2)*T(12)+C(I,3)*T(13)+C(I,14)
      T(16)=P(59)*(T(11)*T(21)+T(12)*T(22)+T(13)*T(23)+T(34))+PS(36)
      P(37)=T(15)/DSQRT(DABS(T(16)))
      IF(DABS(P(37)).GT.P(16))NF(12)=-1
      RETURN
      END
```

```
      SUBROUTINE MEAS6
      DOUBLE PRECISION PS,P,X,C,T,T51,T52,T53
      COMMON PS(72),P(72),X(19),C(19,19),T(19,19),T51(3,3),T52(3,3),
     A       T53(3,3),NF(36)
C     THIS SUBROUTINE IS FOR THE ELEVATION ANGLE, E, FROM THE SECOND C-BAND
C     STATION.
      T(1)=X(1)-PS(38)
      T(2)=X(2)-PS(39)
      T(3)=X(3)-PS(40)
      T(4)=T52(1,1)*T(1)+T52(1,2)*T(2)+T52(1,3)*T(3)
      T(5)=T52(2,1)*T(1)+T52(2,2)*T(2)+T52(2,3)*T(3)
      T(6)=T52(3,1)*T(1)+T52(3,2)*T(2)+T52(3,3)*T(3)
      T(10)=T(4)**2+T(5)**2+T(6)**2
      T(60)=DSQRT(T(4)**2+T(5)**2)
      T(61)=T(10)*T(60)
      T(14)=DATAN2(T(6),T(60))+X(15)
      IF(PS(24).LT.1.D-6)GO TO 100
      T(100)=DSQRT(T(10))
      T(101)=T(6)/T(100)
      T(102)=PS(24)
      T(103)=PS(25)
      CALL REFRAC
      T(14)=T(14)+T(105)
  100 T(15)=P(48)-T(14)
      IF(T(15).GT.2.D0)T(15)=T(15)-6.283185308D0
      T(7)=-(T(4)/T(61))*T(6)
      T(8)=-(T(5)/T(61))*T(6)
      T(9)=T(60)/T(10)
      T(11)=T(7)*T52(1,1)+T(8)*T52(2,1)+T(9)*T52(3,1)
      T(12)=T(7)*T52(1,2)+T(8)*T52(2,2)+T(9)*T52(3,2)
      T(13)=T(7)*T52(1,3)+T(8)*T52(2,3)+T(9)*T52(3,3)
      DO 200 I=1,19
  200 T(I+20)=C(I,1)*T(11)+C(I,2)*T(12)+C(I,3)*T(13)+C(I,15)
      T(16)=P(59)*(T(11)*T(21)+T(12)*T(22)+T(13)*T(23)+T(35))+PS(37)
      P(38)=T(15)/DSQRT(DABS(T(16)))
      IF(DABS(P(38)).GT.P(16))NF(13)=-1
      RETURN
      END
```

OP=1

```
      SUBROUTINE MEAS7
      DOUBLE PRECISION PS,P,X,C,T,T51,T52,T53
      COMMON PS(72),P(72),X(19),C(19,19),T(19,19),T51(3,3),T52(3,3),
     A        T53(3,3),NF(36)
C     THIS SUBROUTINE IS FOR THE RANGE MEASUREMENT, R, FROM THE S-BAND STATION.
      T(1)=X(1)-PS(60)
      T(2)=X(2)-PS(61)
      T(3)=X(3)-PS(62)
      T(100)=DSQRT(T(1)**2+T(2)**2+T(3)**2)
      T(11)=T(1)/T(100)
      T(12)=T(2)/T(100)
      T(13)=T(3)/T(100)
      T(40)=(T(1)*X(4)+T(2)*X(5)+T(3)*X(6))/P(19)
      T(101)=T53(3,1)*T(11)+T53(3,2)*T(12)+T53(3,3)*T(13)
      T(102)=PS(44)
      T(103)=PS(45)
      CALL REFRAC
      T(14)=T(100)-T(40)+T(104)+X(16)
      T(15)=P(49)-T(14)
      DO 100 I=1,19
  100 T(I+20)=C(I,1)*T(11)+C(I,2)*T(12)+C(I,3)*T(13)+C(I,16)
      T(16)=P(59)*(T(11)*T(21)+T(12)*T(22)+T(13)*T(23)+T(36))+PS(56)
      P(39)=T(15)/DSQRT(DABS(T(16)))
      IF(DABS(P(39)).GT.P(16))NF(14)=-1
      RETURN
      END
```

```
      SUBROUTINE MEAS8
      DOUBLE PRECISION PS,P,X,C,T,T51,T52,T53
      COMMON PS(72),P(72),X(19),C(19,19),T(19,19),T51(3,3),T52(3,3),
     A       T53(3,3),NF(36)
C     THIS SUBROUTINE IS FOR THE X ANGLE MEASUREMENT, ALPHA X, FROM THE S-BAND
C     STATION.
      T(1)=X(1)-PS(60)
      T(2)=X(2)-PS(61)
      T(3)=X(3)-PS(62)
      T(4)=T53(1,1)*T(1)+T53(1,2)*T(2)+T53(1,3)*T(3)
      T(5)=T53(2,1)*T(1)+T53(2,2)*T(2)+T53(2,3)*T(3)
      T(6)=T53(3,1)*T(1)+T53(3,2)*T(2)+T53(3,3)*T(3)
      T(10)=T(4)**2+T(5)**2+T(6)**2
      T(61)=DSQRT(T(4)**2+T(5)**2)
      T(100)=DSQRT(T(10))
      T(101)=T(6)/T(100)
      T(102)=PS(44)
      T(103)=PS(45)
      CALL REFRAC
      IF(NF(1).NE.0)GO TO 100
      T(60)=T(4)**2+T(6)**2
      T(14)=DATAN2(T(4),T(6))-T(10)*T(4)*T(105)/(T(61)*T(60))+X(17)
      T(7)=T(6)/T(60)
      T(9)=-T(4)/T(60)
      T(11)=T(7)*T53(1,1)+T(9)*T53(3,1)
      T(12)=T(7)*T53(1,2)+T(9)*T53(3,2)
      T(13)=T(7)*T53(1,3)+T(9)*T53(3,3)
      GO TO 200
  100 T(62)=T(5)**2+T(6)**2
      T(14)=DATAN2(-T(5),T(6))+T(10)*T(5)*T(105)/(T(61)*T(62))+X(17)
      T(8)=-T(6)/T(62)
      T(9)=T(5)/T(62)
      T(11)=T(8)*T53(2,1)+T(9)*T53(3,1)
      T(12)=T(8)*T53(2,2)+T(9)*T53(3,2)
      T(13)=T(8)*T53(2,3)+T(9)*T53(3,3)
  200 T(15)=P(50)-T(14)
      IF(T(15).GT.2.D0)T(15)=T(15)-6.283185308D0
      DO 300 I=1,19
```

OP=1

```
 300 T(I+20)=C(I,1)*T(11)+C(I,2)*T(12)+C(I,3)*T(13)+C(I,17)
     T(16)=P(59)*(T(11)*T(21)+T(12)*T(22)+T(13)*T(23)+T(37))+PS(57)
     P(40)=T(15)/DSQRT(DABS(T(16)))
     IF(DABS(P(40)).GT.P(16))NF(15)=-1
     RETURN
     END
```

```
      SUBROUTINE MEAS9
      DOUBLE PRECISION PS,P,X,C,T,T51,T52,T53
      COMMON PS(72),P(72),X(19),C(19,19),T(19,19),T51(3,3),T52(3,3),
     A       T53(3,3),NF(36)
C     THIS SUBROUTINE IS FOR THE Y ANGLE MEASUREMENT, ALPHA Y, FROM THE S-BAND
C     STATION.
      T(1)=X(1)-PS(60)
      T(2)=X(2)-PS(61)
      T(3)=X(3)-PS(62)
      T(4)=T53(1,1)*T(1)+T53(1,2)*T(2)+T53(1,3)*T(3)
      T(5)=T53(2,1)*T(1)+T53(2,2)*T(2)+T53(2,3)*T(3)
      T(6)=T53(3,1)*T(1)+T53(3,2)*T(2)+T53(3,3)*T(3)
      T(10)=T(4)**2+T(5)**2+T(6)**2
      T(100)=DSQRT(T(10))
      T(101)=T(6)/T(100)
      T(102)=PS(44)
      T(103)=PS(45)
      CALL REFRAC
      IF(NF(1).NE.0)GO TO 100
      T(60)=DSQRT(T(4)**2+T(6)**2)
      T(61)=T(10)*T(60)
      T(62)=T(5)/DSQRT(T(4)**2+T(5)**2)
      T(63)=T(6)/T(60)
      T(14)=DATAN2(T(5),T(60))-T(62)*T(63)*T(105)+X(18)
      T(7)=-(T(4)/T(61))*T(5)
      T(8)=T(60)/T(10)
      T(9)=-(T(6)/T(61))*T(5)
      GO TO 200
  100 T(64)=DSQRT(T(5)**2+T(6)**2)
      T(65)=T(10)*T(64)
      T(66)=T(4)/DSQRT(T(4)**2+T(5)**2)
      T(67)=T(6)/T(64)
      T(14)=DATAN2(T(4),T(64))-T(66)*T(67)*T(105)+X(18)
      T(7)=T(64)/T(10)
      T(8)=-(T(5)/T(65))*T(4)
      T(9)=-(T(6)/T(65))*T(4)
  200 T(11)=T(7)*T53(1,1)+T(8)*T53(2,1)+T(9)*T53(3,1)
      T(12)=T(7)*T53(1,2)+T(8)*T53(2,2)+T(9)*T53(3,2)
```

```
      T(13)=T(7)*T53(1,3)+T(8)*T53(2,3)+T(9)*T53(3,3)
      T(15)=P(51)-T(14)
      IF(T(15).GT.2.D0)T(15)=T(15)-6.28318530800
      DO 300 I=1,19
  300 T(I+20)=C(I,1)*T(11)+C(I,2)*T(12)+C(I,3)*T(13)+C(I,18)
      T(16)=P(59)*(T(11)*T(21)+T(12)*T(22)+T(13)*T(23)+T(38))+PS(58)
      P(41)=T(15)/DSQRT(DABS(T(16)))
      IF(DABS(P(41)).GT.P(16))NF(16)=-1
      RETURN
      END
```

OP=1

```
      SUBROUTINE MEAS10
      DOUBLE PRECISION PS,P,X,C,T,T51,T52,T53
      COMMON PS(72),P(72),X(19),C(19,19),T(19,19),T51(3,3),T52(3,3),
     A       T53(3,3),NF(36)
C     THIS SUBROUTINE IS FOR THE DOPPLER MEASUREMENT, N CYCLES, FROM THE S-BAND
C     STATION.
      T(1)=X(1)-PS(60)
      T(2)=X(2)-PS(61)
      T(3)=X(3)-PS(62)
      T(100)=DSQRT(T(1)**2+T(2)**2+T(3)**2)
      T(60)=PS(67)/T(100)
      T(101)=(T53(3,1)*T(1)+T53(3,2)*T(2)+T53(3,3)*T(3))/T(100)
      T(102)=PS(44)
      T(103)=PS(45)
      CALL REFRAC
      T(40)=(T(1)*X(4)+T(2)*X(6)+T(3)*X(7))/P(19)
      T(14)=PS(63)*P(32)*P(2)-PS(67)*(T(100)+T(104)-T(40))+X(19)
      P(32)=P(32)+1.D0
      T(15)=P(52)-T(14)
      T(11)=-T(60)*T(1)
      T(12)=-T(60)*T(2)
      T(13)=-T(60)*T(3)
      DO 100 I=1,19
  100 T(I+20)=C(I,1)*T(11)+C(I,2)*T(12)+C(I,3)*T(13)+C(I,19)
      T(16)=3.D0*(T(11)*T(21)+T(12)*T(22)+T(13)*T(23)+T(39))+PS(59)
      P(42)=T(15)/DSQRT(DABS(T(16)))
      IF(DABS(P(42)).GT.P(16))NF(17)=-1
      RETURN
      END
```

13. MODIFICATION FOR THE REAL-TIME PROGRAM

NASA/JSC has made several modifications of the program outlined in this document. Perhaps the most important modification is a change in the definition of six of the nineteen state variables. In an effort to speed up the program, when propagating the error covariance matrix ahead in time, the following state variables have been changed.

$$\underline{\dot{R}}_{EF} \text{ is replaced with } \underline{\dot{R}}_{EF}\,\Delta T.$$

$$\underline{\ddot{R}}_{EF} \text{ is replaced with } \underline{\ddot{R}}_{EF}\,\Delta T^2/2.$$

The new equations of motion are

$$\underline{R}_{EF,i} = \underline{R}_{EF,i-1} + (\underline{\dot{R}}_{EF}\,\Delta T)_{i-1} + (\underline{\ddot{R}}_{EF}\,\Delta T^2/2)_{i-1}$$

$$(\underline{\dot{R}}_{EF}\Delta T)_i = (\underline{\dot{R}}_{EF}\Delta T)_{i-1} + 2(\underline{\ddot{R}}_{EF}\,\Delta T^2/2)_{i-1}$$

$$(\underline{\ddot{R}}_{EF}\Delta T^2/2)_i = \exp(-\Delta T/\tau_a)(\underline{\ddot{R}}_{EF}\Delta T^2/2)_{i-1}$$

The first nine elements of the state noise vector become

$$\underline{S} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ (\sigma_a \Delta T^2/2)\sqrt{1 - \exp(-2\Delta T/\tau_a)}\ \eta_{a1} \\ (\sigma_a \Delta T^2/2)\sqrt{1 - \exp(-2\Delta T/\tau_a)}\ \eta_{a2} \\ (\sigma_a \Delta T^2/2)\sqrt{1 - \exp(-2\Delta T/\tau_a)}\ \eta_{a3} \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

Thus the standard deviation of the acceleration state noise is scaled by $\Delta T^2/2$. Let $\sigma_a = 6\ \text{m/sec}^2$ and $\Delta T = 0.2$ second. Then the new acceleration state noise σ is

$$\sigma_{a'} = \sigma_a\ \Delta T^2/2 = .12\ \text{meters}$$

$$= 1.88 \cdot 10^{-8}\ \text{er}$$

The initial velocity variance is also scaled by ΔT^2. We had been using an initial velocity variance of $(8000\ \text{m/sec})^2 = 20.4\ \text{er}^2/\text{hr}^2$. The new initial

velocity variance becomes $(8000\ \Delta T\ m)^2 = (1600\ m)^2 = 6.3 \cdot 10^{-8}\ er^2$. The position variances are, of course, unchanged.

Note also that the speed of light correction term in the measurement equations, $\rho\dot{\rho}/c$, was

$$\rho\dot{\rho}/c = \underline{R}_{V/A,EF} \cdot \dot{\underline{R}}_{V/A,EF}/c$$

It becomes

$$\rho\dot{\rho}/c = \underline{R}_{V/A,EF} \cdot (\dot{\underline{R}}_{V/A,EF}\ \Delta T)/(c\Delta T)$$

It is estimated that the new state vector will reduce the Kalman filter cycle times by about 10%.

REFERENCES

1. Lear, William M: Mathematical Models for Short Range Data From a Single S-Band Station. JSC IN 76-FM-66, Aug. 25, 1976.

2. Lear, William M: The Multi-Phase Navigation Program for the Space Shuttle. JSC IN 73-FM-132, Sept. 8, 1973.

3. Lear, William M: The High-Speed Tracking Data Processor for the Space Shuttle (TRW Bench Program). TRW Technical Report No. 30759-6002-RU-00, Mar. 31, 1977.

END
DATE
Filmed
10-12-77